Double-exponential lower bounds for graph identification problems

Florent Foucaud
joint works with:

Esther Galby, Liana Khazaliya, Shaohua Li, Fionn Mc Inerney, Roohani Sharma, Prafullkumar Tale (ICALP 2024)

Dipayan Chakraborty, Diptapriyo Majumdar, Prafullkumar Tale (ISAAC 2024)

UNIVERSITÉ
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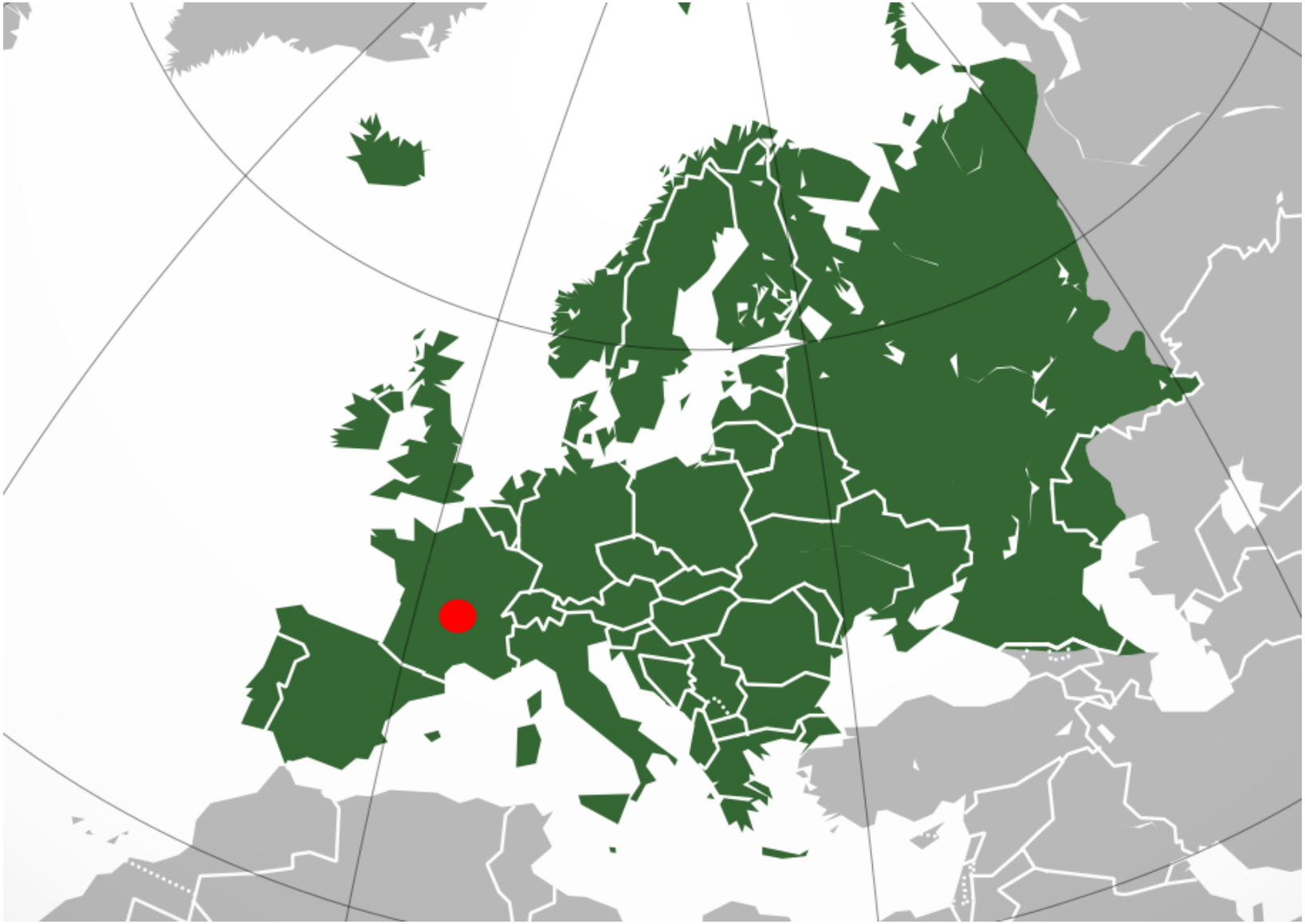
IWOCA 2026 in Clermont-Ferrand

- 37th International Workshop on Combinatorial Algorithms.
- Yearly since 1989, started in Australia.
- Submission deadline: January 26 (abstract), February 2 (full paper)
- Conference dates: June 8-12, 2026
- Collaborative aspect
- Website: `https://iwoca2026.limos.fr`

Where: Clermont-Ferrand, France



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MICHELIN


Saint
Nectaire

(START OF THE TALK)

Treewidth

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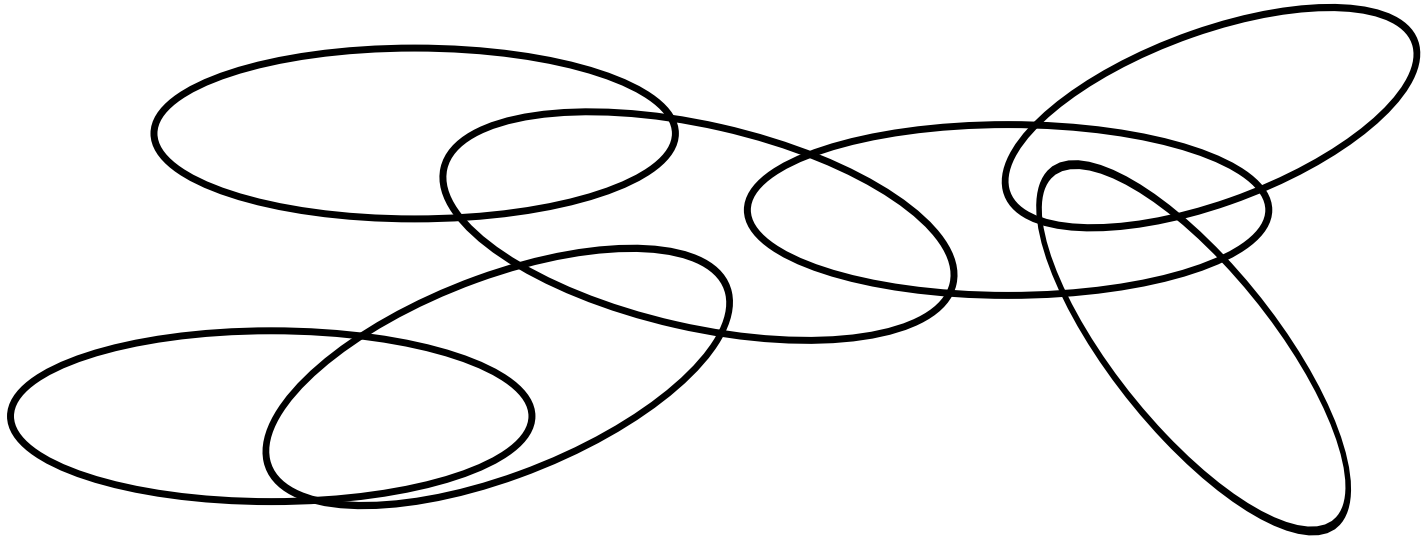
Graph G of treewidth k: looks like a tree where edges are replaced by "bags" of size k.



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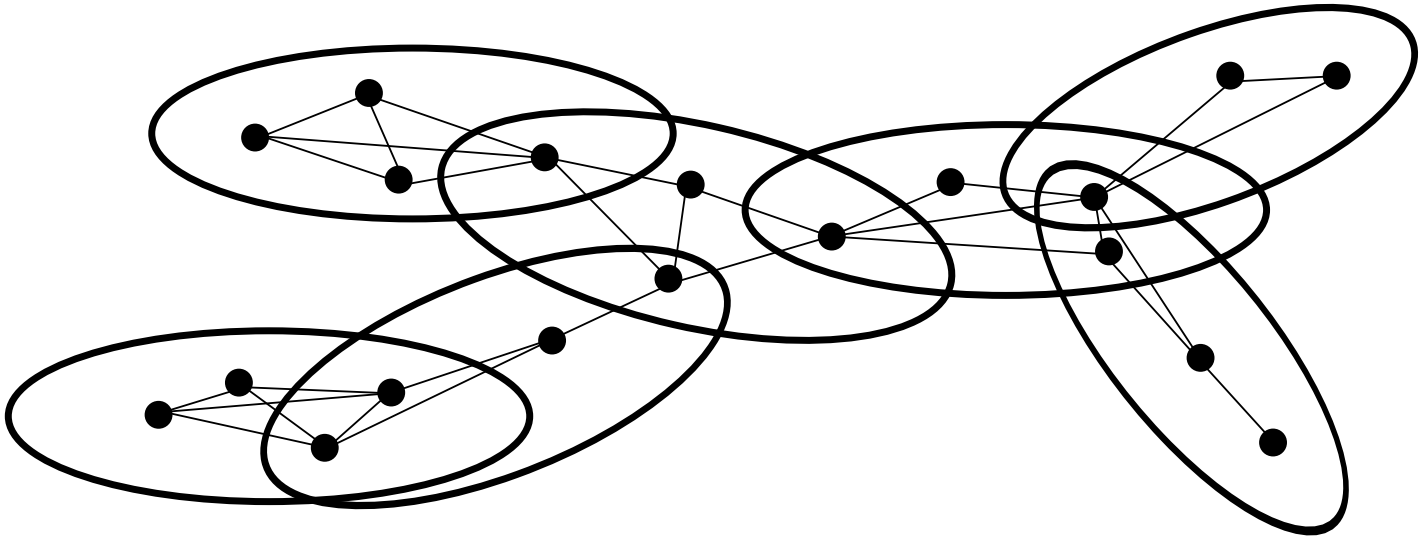

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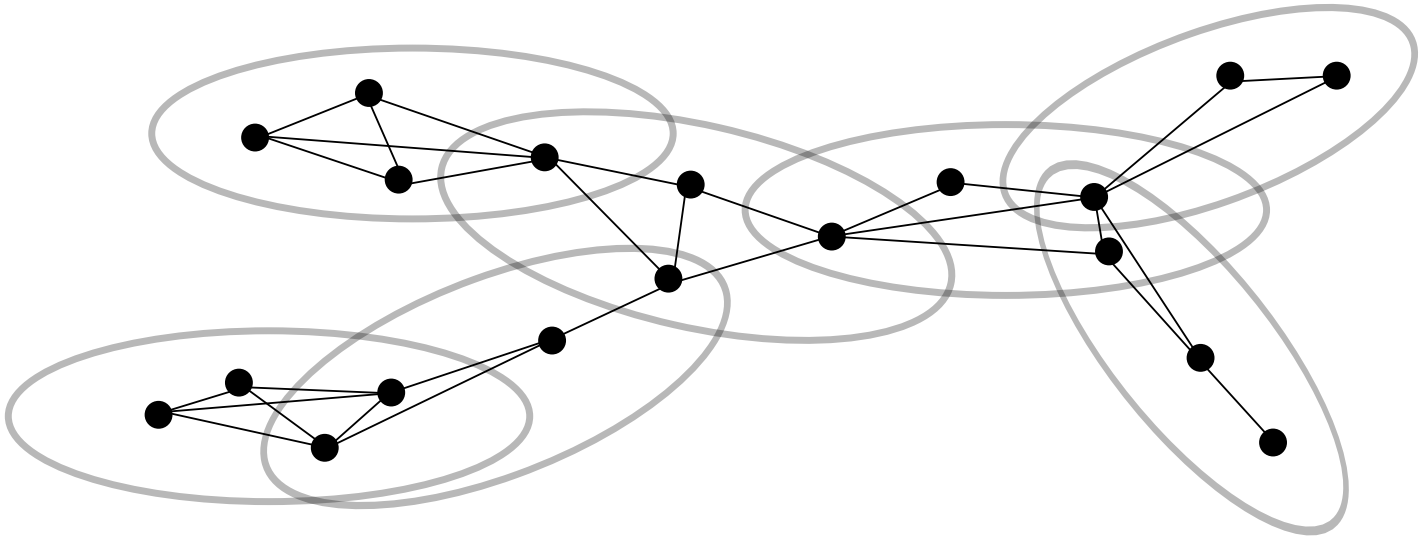

Informally:

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- its width is the largest size of a bag
- **treewidth** $\mathtt{tw}(G)$ of G is the smallest width over all tree-decompositions

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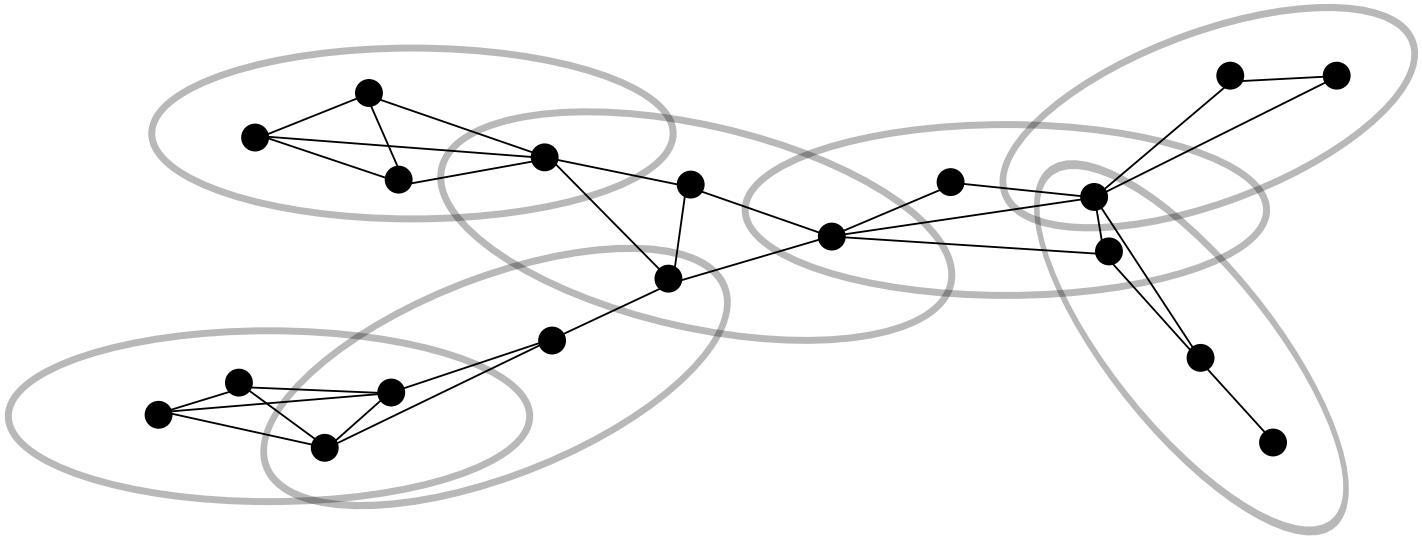

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Very important in:

- Structural graph theory
- Graph algorithms
- Many applications in other areas: Database queries, Optimization, Boolean SAT...

Treewidth: the **King** of Structural Parameters

Fixed parameter tractable (FPT) problems

A **decision problem** with input $\mathcal{I}$ and **parameter** k is FPT parameterized by k if it can be solved in time $f(k) \cdot |\mathcal{I}|^{O(1)}$, where f is a computable function.

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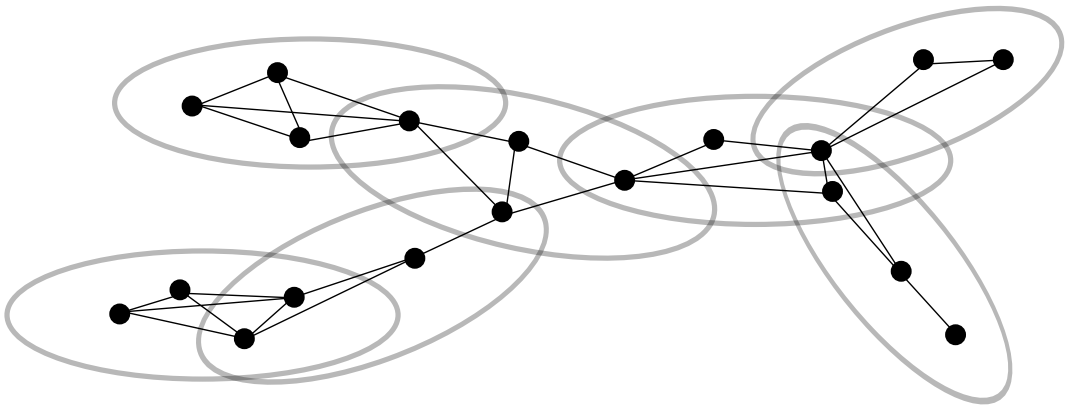

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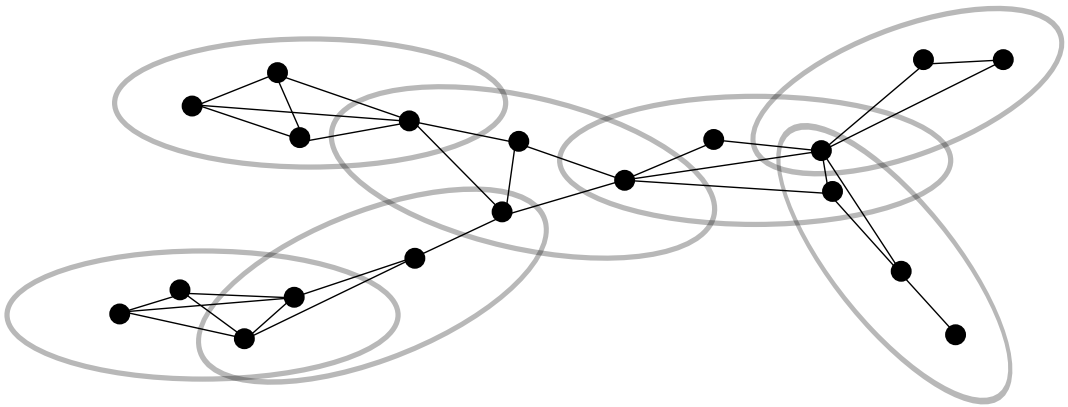

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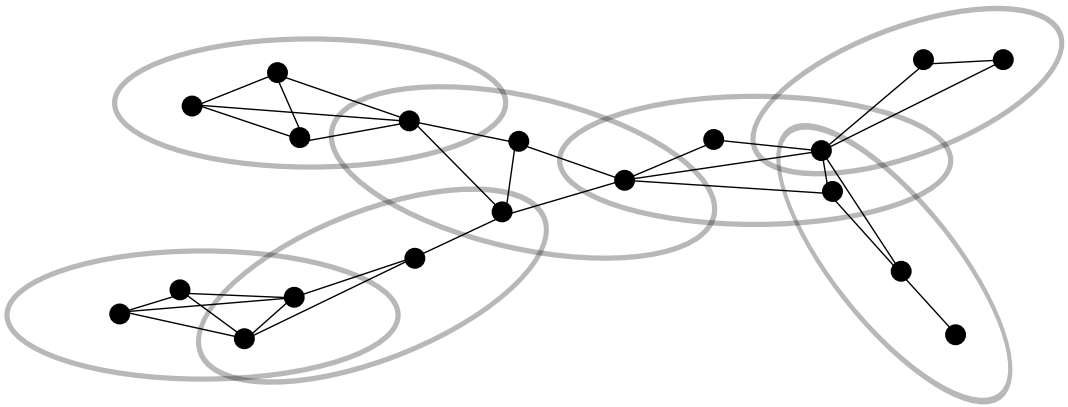

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However, $f(\mathtt{tw})$ may be a tower of exponentials!

ETH-based conditional lower bounds on $f(\mathtt{tw})$ for FPT algorithms

Exponential Time Hypothesis (ETH) [Impagliazzo, Paturi, 1990]

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Rarer results: Assuming the ETH,

- QSAT (PSPACE-complete) with k alternations admits a lower bound of a tower of exponents of height k in the tw of the primal graph [Fichte, Hecher, Pfandler, 2020];
- k-Choosability (Π_2^p-complete) and k-Choosability Deletion (Σ_3^p-complete) admit double- and triple-exponential lower bounds in tw, resp. [Marx, Mitsou, 2016];
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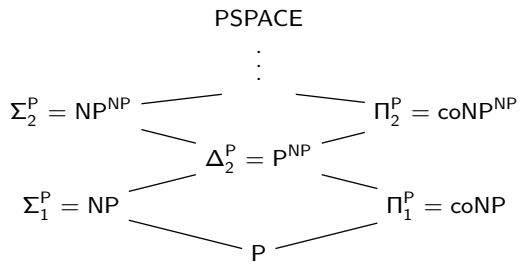

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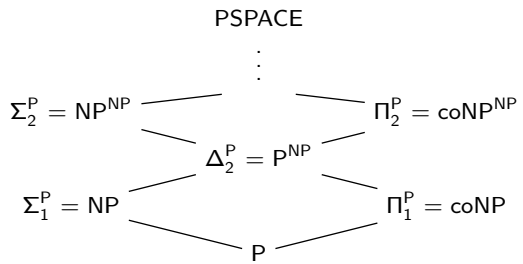

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Common theme: complexity classes higher than NP require large dependency in $\mathtt{tw}$.

Our results

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ETH-based double-exponential lower bounds in the treewidth for NP-complete problems.

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We develop a **technique** to prove such lower bounds for (nice) graph problems:

Theorem [F., Galby, Khazaliya, Li, Mc Inerney, Sharma, Tale, ICALP 2024]

Metric Dimension and Geodetic Set

- can be solved in $2^{\mathtt{diam}^{O(\mathtt{tw})}} \cdot n^{O(1)}$ time
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Locating-Dominating Set and Test Cover

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Identification problems

A graph identification problem

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want to **distinguish** all elements of a discrete structure using a (small) solution set

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Examples: Test Cover, Separating Set/System, Identifying Code...

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Locating-dominating set [Slater, 1980's]

$D \subseteq V$ **locating-dominating set** of $G = (V, E)$:

- for every $v \in V$, $N[v] \cap D \neq \emptyset$ (domination)
- $\forall u \neq v$ of $V \setminus D$, $N(u) \cap D \neq N(v) \cap D$ (location)

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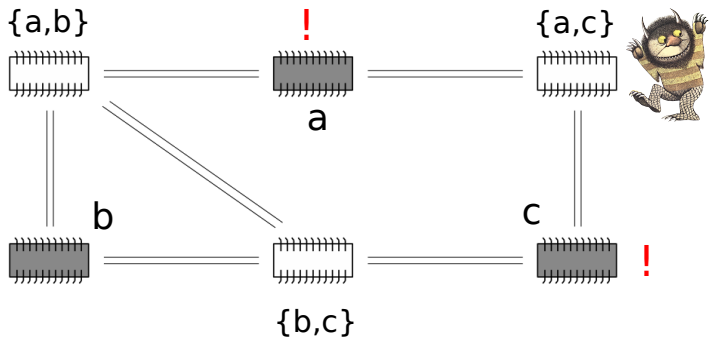
Motivation: fault-detection in networks.

$\rightarrow$ The set D of grey processors is a set of fault-detectors.



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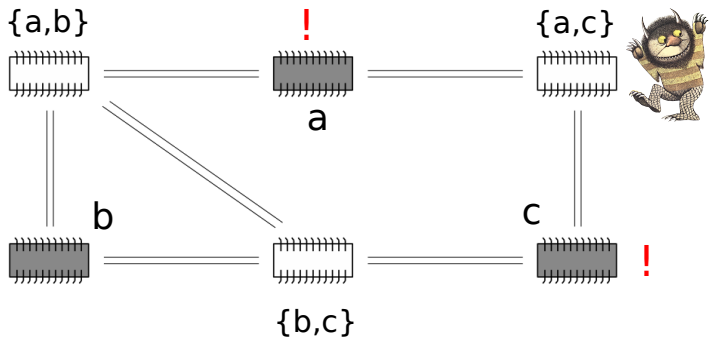

Other applications:

biological testing, graph isomorphism, machine learning (VC-dimension)...

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Decision problem:

Locating-Dominating Set

Input: an undirected graph $G = (V, E)$ and an integer $k \geq 1$
Question: Does G have a locating-dominating set of size at most k?

NP-complete, FPT for treewidth using MSOL and Courcelle's theorem

Proof ideas

Main result for Locating-dominating Set

Theorem [Chakraborty, F., Majumdar, Tale, ISAAC 2024]

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Algorithm: classic **dynamic programming** over a tree-decomposition.

Main idea: for each bag, remember which **subsets** of solution vertices form the intersection of some neighborhood

$\rightarrow$ Number of DP-states per bag: $2^{2^{O(\mathtt{tw})}}$ possible collections of subsets

Main result for Locating-dominating Set: lower bound

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Roughly, n-variable 3-SAT cannot be solved in time $2^{o(n)}$.

Goal: reduction from n-variable 3-SAT such that $\mathtt{tw}(G) = O(\log n)$.

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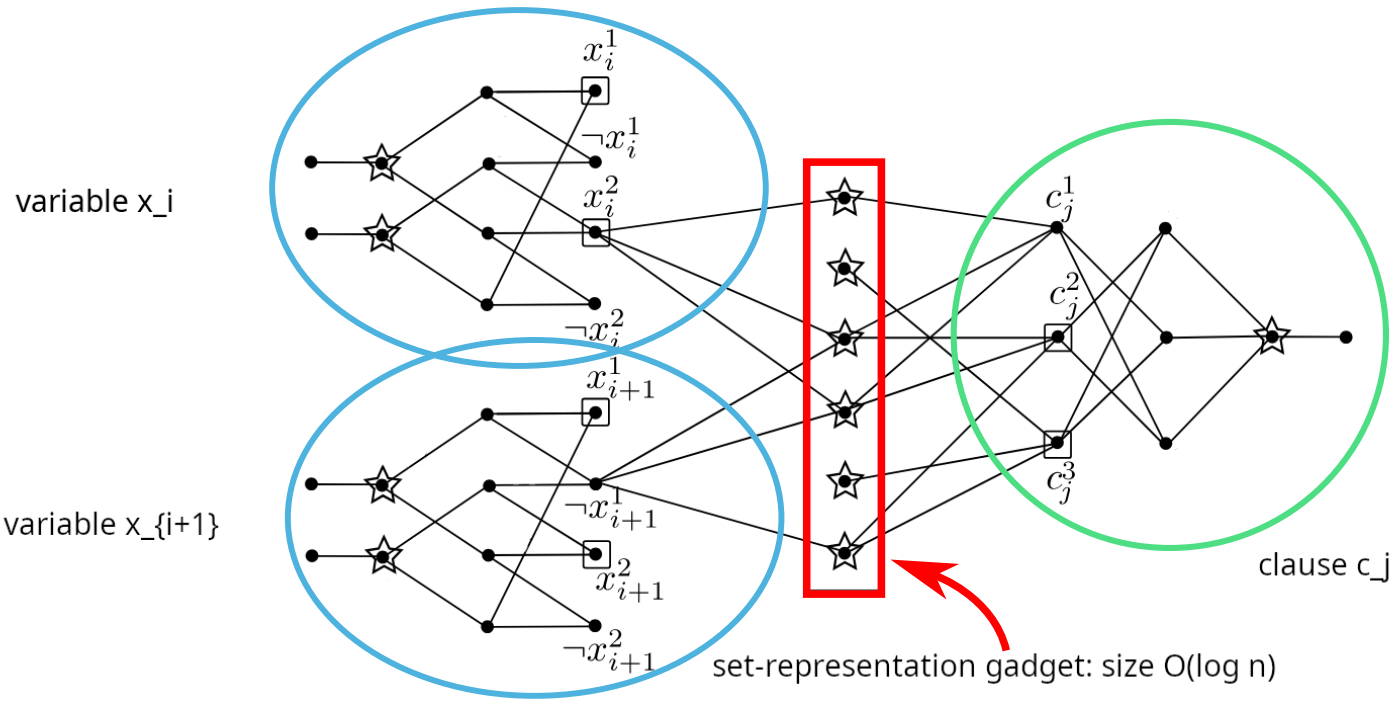
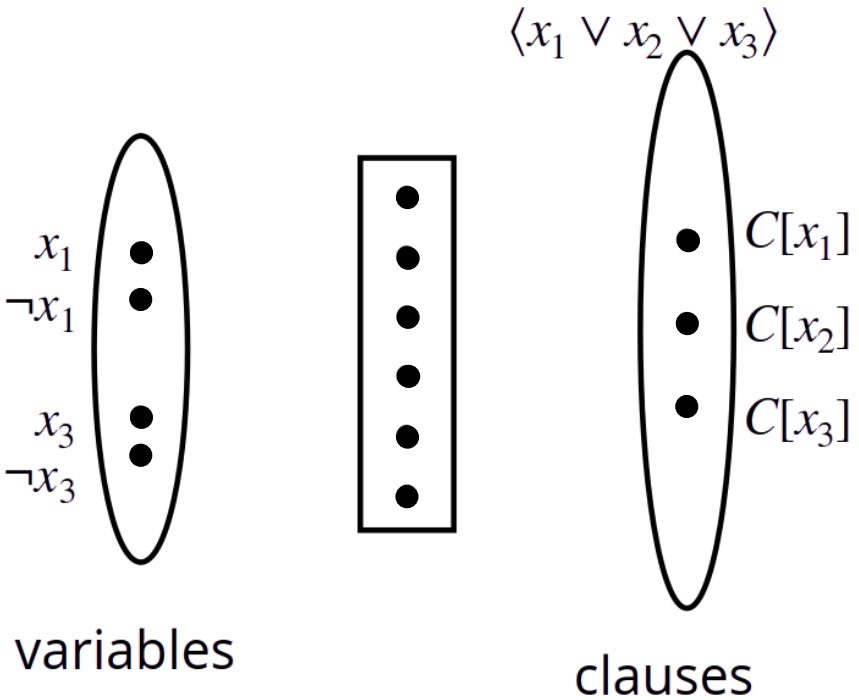


How to encode clause-variable relations using a set of size $O(\log n)$?

Set-representation gadget: vertex set S of size $2p$.



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Set-representation gadget: vertex set S of size $2p$. Assign as neighbours, a **distinct subset** of size p to **each literal occurrence**. $\binom{2p}{p} \geq 4n \rightarrow p = O(\log n)$

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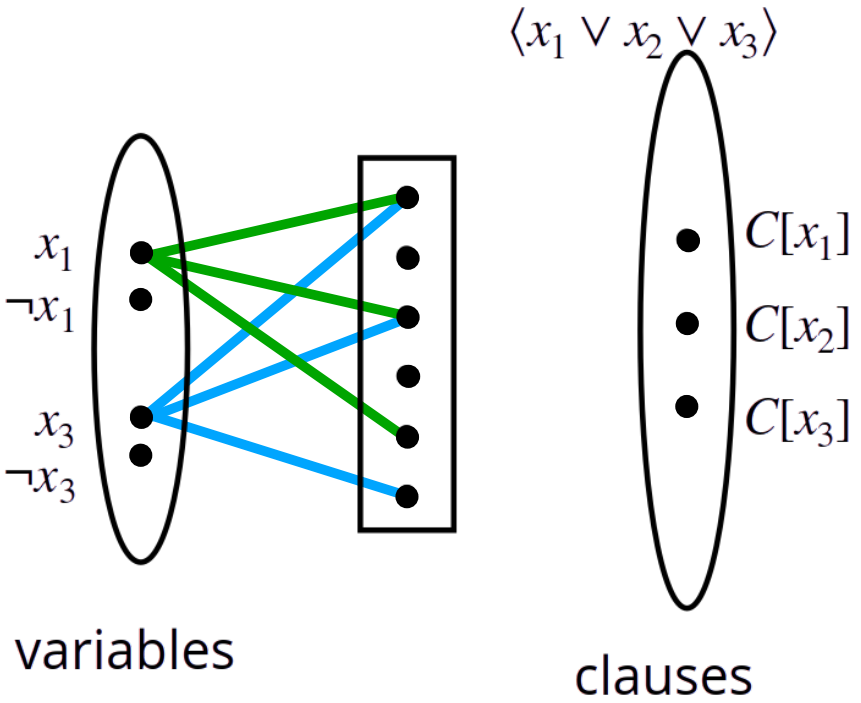

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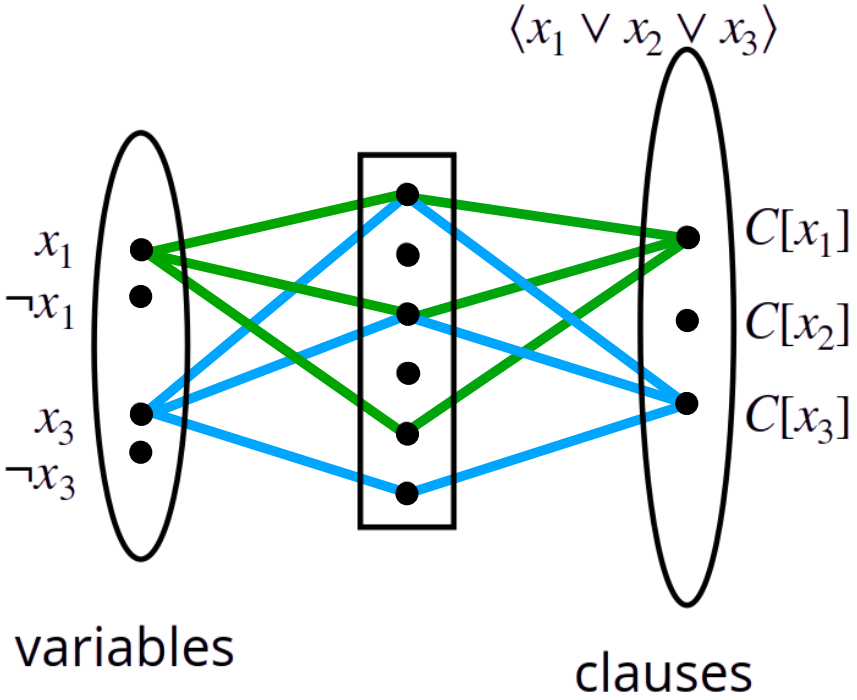

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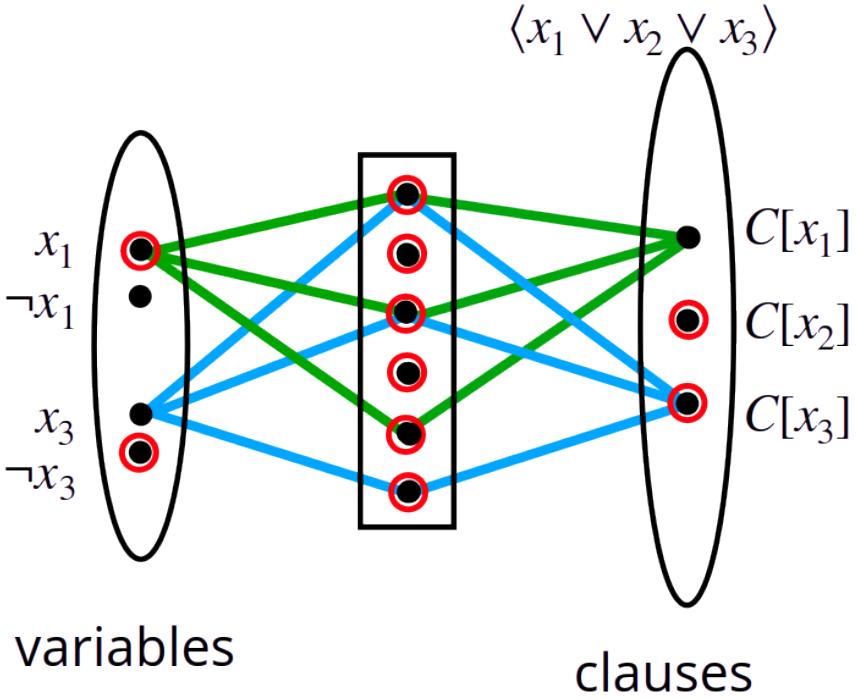

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Metric dimension of a graph $G = (V, E)$ [Slater '75 + Harary, Melter '76]

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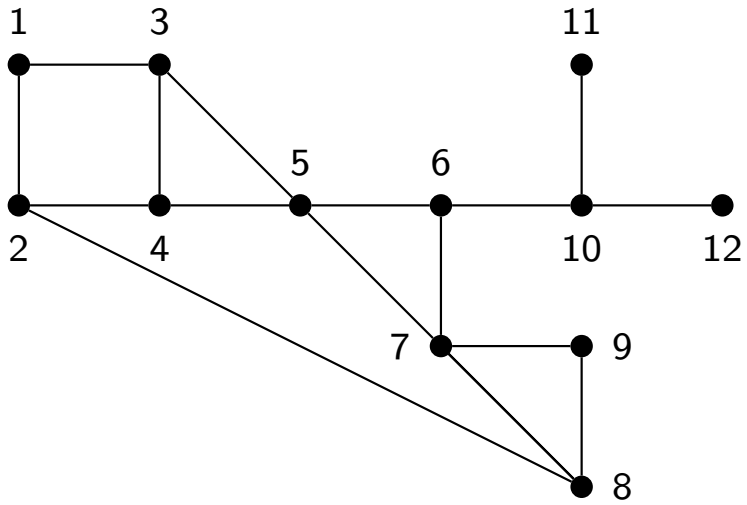

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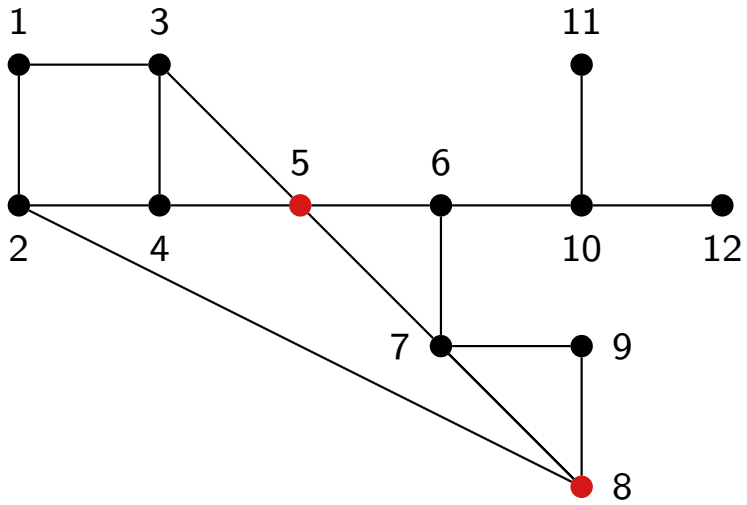

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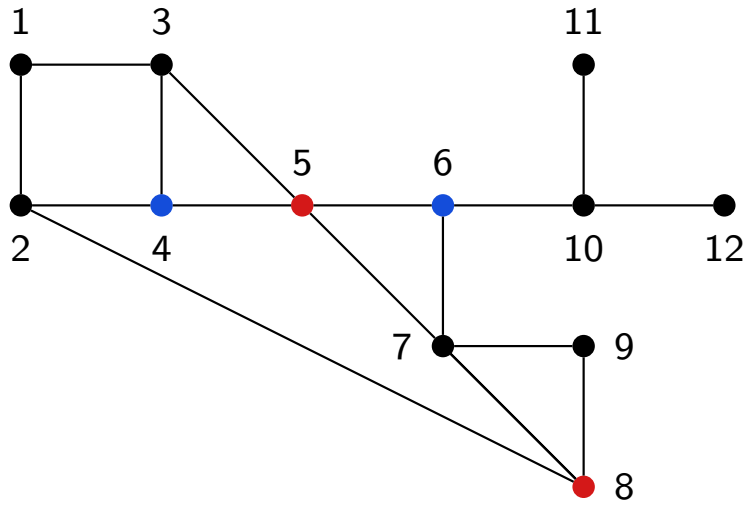
$S \subseteq V$ is a resolving set of G if $\forall u, v \in V$, $\exists z \in S$ with $d(z, u) \neq d(z, v)$. The **minimum size** of a resolving set of G is the metric dimension of G.



Vertices 4 and 6 are **not** resolved by 5 nor 8.

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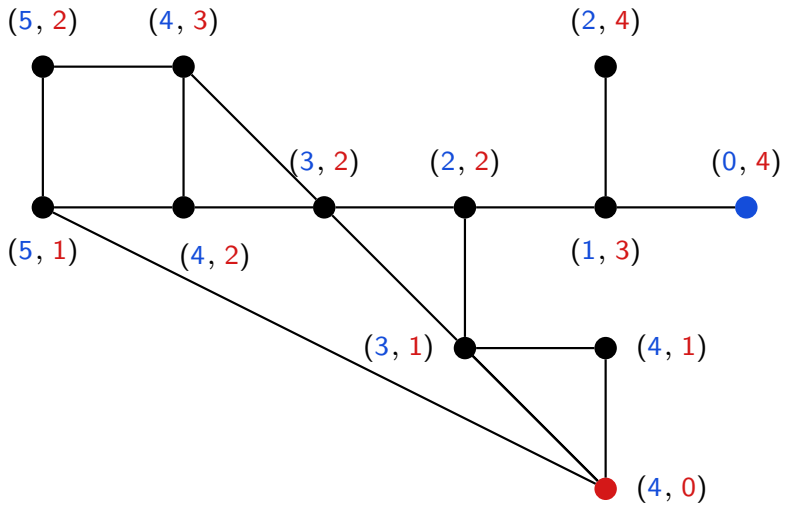

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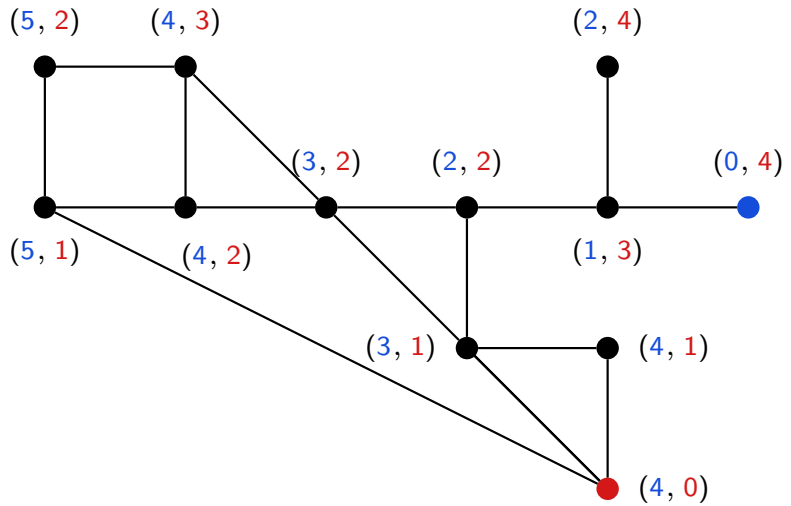

Metric Dimension

Input: an undirected graph $G = (V, E)$ and an integer $k \geq 1$
Question: Is the metric dimension of G at most k?

Results for Metric Dimension

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Theorem [F., Galby, Khazaliya, Li, Mc Inerney, Sharma, Tale, ICALP 2024]

Metric Dimension:

- can be solved in $2^{\mathtt{diam}^{O(\mathtt{tw})}} \cdot n^{O(1)}$ time
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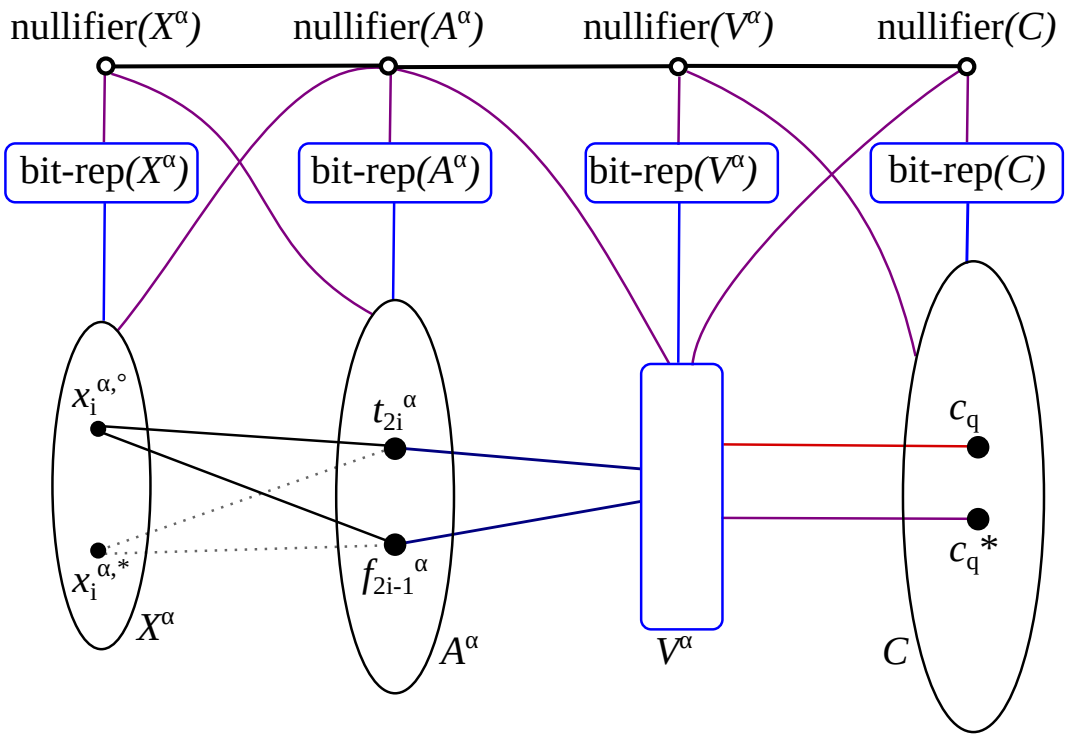
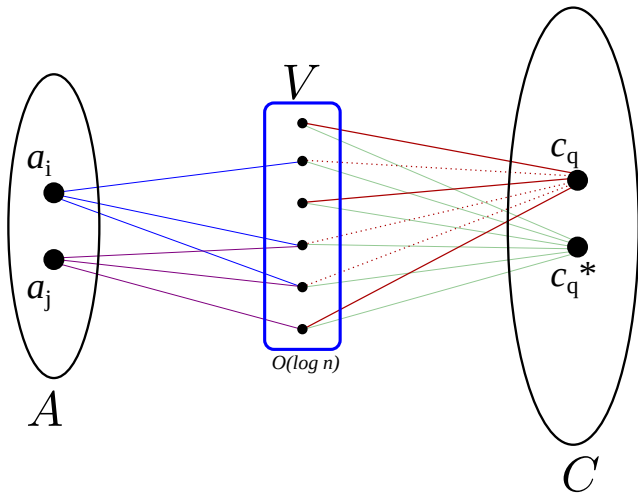
The original proof is a more complex use-case of the set-representation gadget:





Conclusion

Main takeaway:

- Most natural NP-complete problems FPT for $\mathtt{tw}$ can be solved in time $2^{poly(\mathtt{tw})} \cdot n^{O(1)}$
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Further results:

- These problems have other "exotic" behaviours
- Other NP-complete problems behave similarly:
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THANKS!