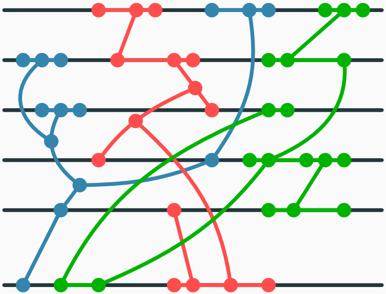
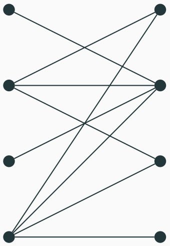
Tight Erdős-Pósa bounds for minors

Jean-Florent Raymond
(TU Berlin)



Joint work with Wouter Cames van Batenburg, Tony Huynh, and Gwenaël Joret (Université Libre de Bruxelles).

Packing and covering in bipartite graphs



Max. number
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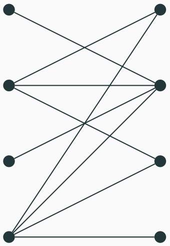

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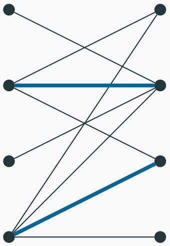

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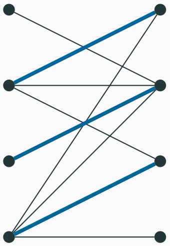

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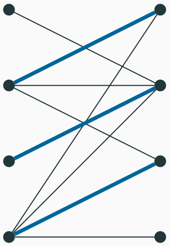

$\text{pack}_{K_2} = 3$

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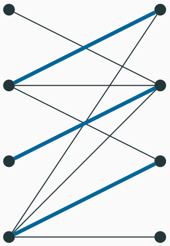

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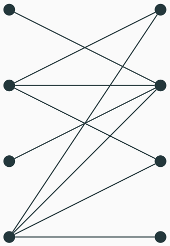

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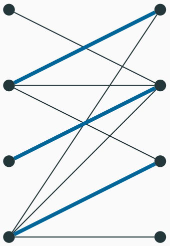

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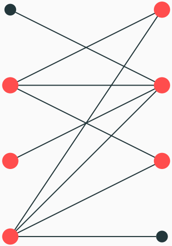

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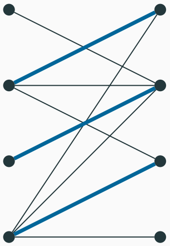

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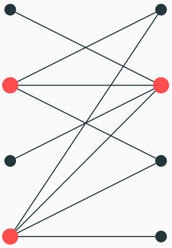

Packing and covering in bipartite graphs

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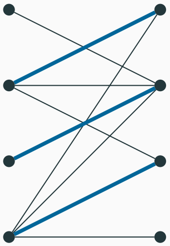

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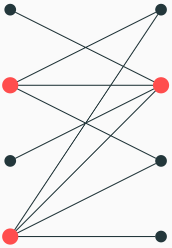

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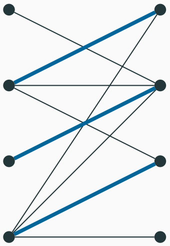

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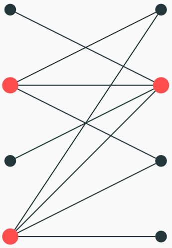

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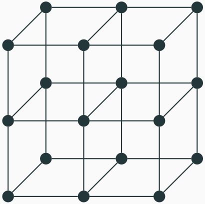
$\text{cover} = \text{pack}$

(Kőnig's Theorem, 1931)



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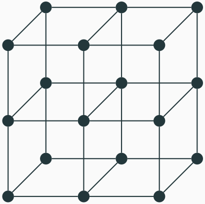

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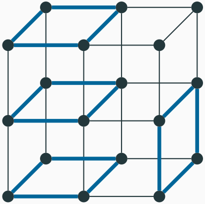

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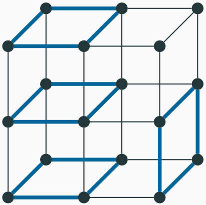

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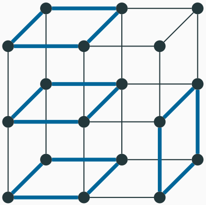

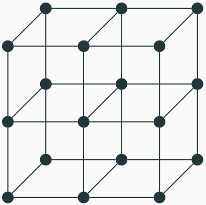

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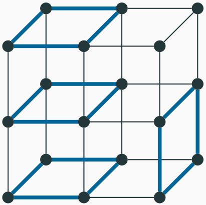

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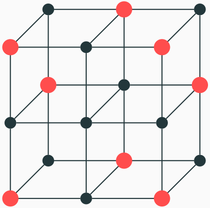

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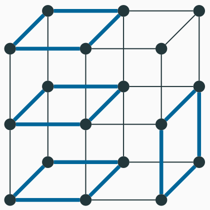

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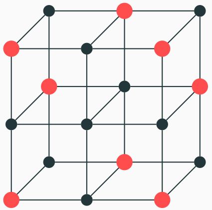

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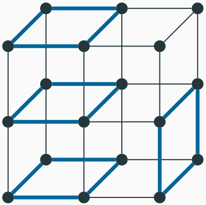
Max. number of disjoint cycles?

Min. number of vertices to cover all cycles?



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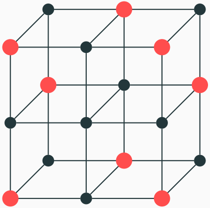

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$$\text{pack} \leqslant \text{cover} \leqslant c \cdot \text{pack} \log \text{pack}$$

(Erdős-Pósa Theorem, 1965)

Theorem (Erdős and Pósa, Can. J. Math. 1965)

Every graph has one of the following:

- k *vertex-disjoint cycles;*
- *a feedback vertex set of size* $O(k \log k)$.

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Min-max theorem (like Kőnig's and Menger's theorems, etc.).

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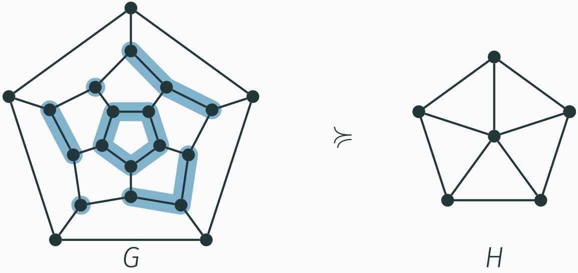
Our goal: generalize from cycles to minor-models.

Definition

An *H*-model in G is a set $\{S_u\}_{u \in V(H)}$ of disjoint subsets of $V(G)$ s.t.

- the $G[S_u]$'s are connected;
- edge uv in $H \Rightarrow$ edge between S_u and S_v in G.



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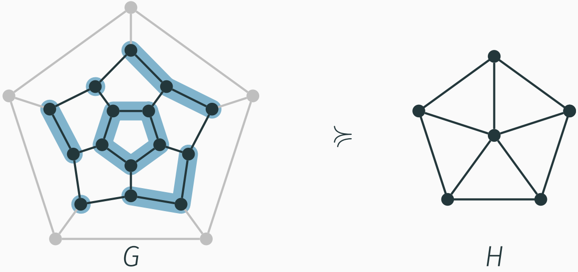

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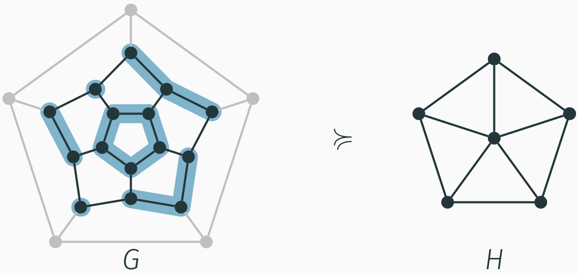

G has a H-model $\iff$ H is a minor of G

Definition

H has the Erdős-Pósa property if there is a function f s.t., for every graph G and $k \in \mathbb{N}$,

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With which gap?

Graph H	EP gap	Reference	
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A non-exhaustive history of Erdős-Pósa gaps

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- H not planar $\Rightarrow$ no Erdős-Pósa property;
- H has a cycle $\Rightarrow$ no $o(k \log k)$ gap.

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Lemma (Cames van Batenburg, Huynh, Joret, R., 2018+)

For every graph G and every planar graph H,

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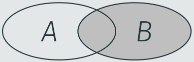
- $G =$ 

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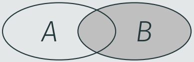
- $G =$ 

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The constant hidden in the "O" notation depends on:

- the graph H;
- the definition of "large".

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Goal: “G has a *small* H-model or a large *useless* part”

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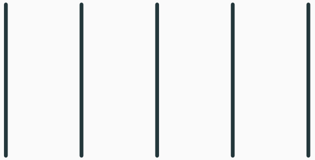
Maximum collection of disjoint paths of length ℓ:

(covering *G*, for simplicity)



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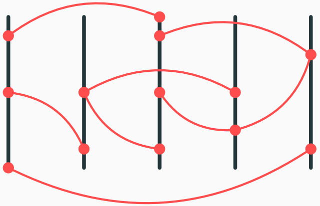

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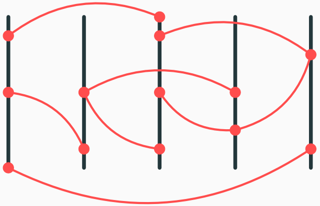

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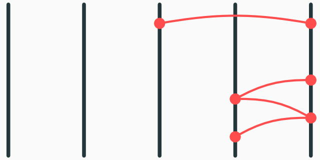

- either every path sees $\geqslant 3$ other paths:

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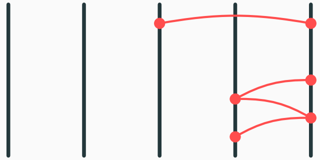

cycle of length $\leqslant 2\ell$

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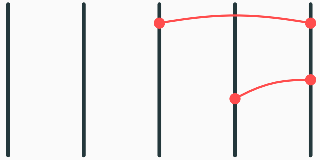

there are $\leqslant 2$ incident edges

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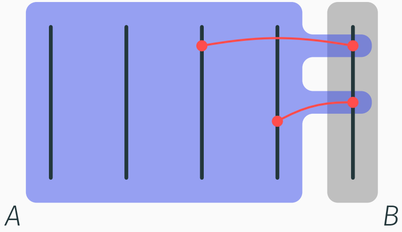

- B is K_3-minor free
- $|B| \geqslant \text{large}(|A \cap B|)$

- either every path sees $\geqslant 3$ other paths:
  cycle of length $O(\ell \cdot \log |G|)$
- or one path sees $\leqslant 2$ other paths

Proof sketch for $H = K_3$

Goal: "G has a *small* H-model or a large *useless* part"

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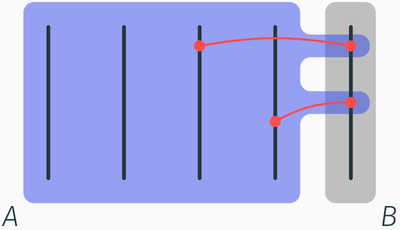

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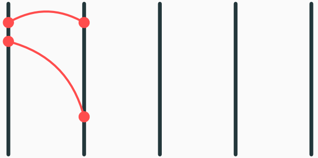
  cycle of length $\leqslant 2\ell$ or large useless part.

How to generalize?



Crucial property: we can conclude when two paths are connected with many edges.

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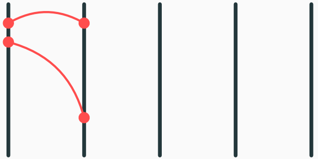

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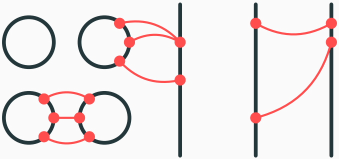
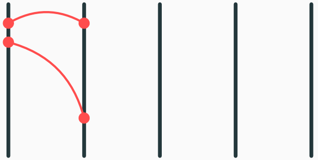
Possible extension to $H = K_4$:



Pack cycles of bounded size first, then paths.



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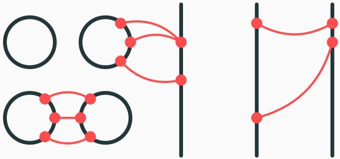
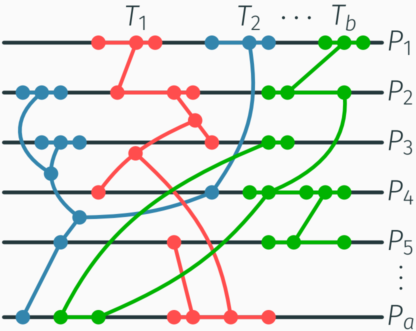


Pack cycles of bounded size first, then paths.

$\rightsquigarrow$ gap $O(k \log k)$ when H is a wheel
(Aboulker, Fiorini, Huynh, Joret, R. and Sau, 2018)

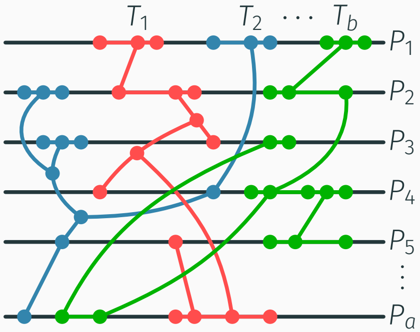

T_1
T_2 ⋯ T_b
P_1
P_2
P_3
P_4
P_5
⋮
P_a

An $a \times b$-orchard in G consists in collections

- $P_1, \dots, P_a$ of vertex-disjoint (horizontal) paths; and
- $T_1, \dots, T_b$ of vertex-disjoint (vertical) trees,

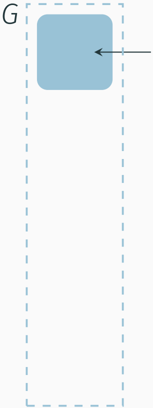
s.t. for every $i \in [a]$, $j \in [b]$:

- $P_i \cap T_j \neq \emptyset$ and connected; and
- each leaf of T_j lies on some horizontal path.

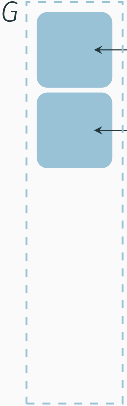


Decomposition into orchards

G



max. collection of disjoint $m \times \omega(m)$-orchards in G



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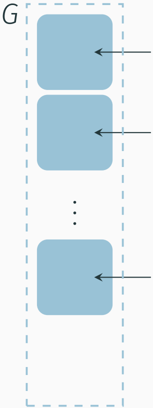
max. collection of disjoint $(m-1) \times \omega(m-1)$-orchards in what remains



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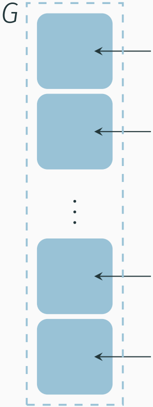

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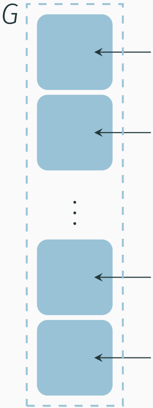
leftovers



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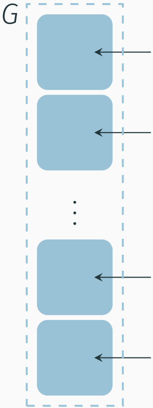

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- many edges between two orchards $\Rightarrow$ small model or *better* decomposition
- only few edges between two orchards $\Rightarrow$ small separation

Consequences

$\mathsf{pack}_H(G)$ max. number of disjoint H-models in G
$\mathsf{cover}_H(G)$ min. size of a cover of H-models in G

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CONSEQUENCE 1/4: ALGORITHMS

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For every planar graph H, there is a polytime $O(\log(\text{OPT}))$-approximation algorithm for pack_H.

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(idem for cover_H, but $O(1)$-approximations are already known)

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Consequence 3/4: Packing cycles with modularity constraints

Theorem (Thomassen, JGT 1988)

For every $m \in \mathbb{N}_{\geqslant 1}$ there is a function f s.t., for every $k \in \mathbb{N}$ and every graph G,

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- from our result: $f(k) = O(k \log k)$ (tight)

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The previous theorem also follows from our results.

Open problems

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H planar $\Rightarrow$ *there is a* $O(k \log k)$ *gap for H.*

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Same behavior?

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	edge	?	?
topo. minors	vertex	✓?	?
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Thank you for your attention!