From Problem Solving to Research Thinking: A Graph-Theoretic Whodunit*

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Abstract. Popularizing research thinking methods is critical in a society where skepticism towards science has grown throughout the years. Due to the dramatic spread of generative AI systems, the need for such popularization in Computer Science (CS) has become more and more important. We propose an activity aiming at demystifying the methodology used by CS researchers through a whodunit puzzle based on the novel "Who killed the Duke of Densmore?" by Claude Berge. After introducing the problem to participants, along with the definition of interval graphs and some of their basic properties, we invite the participants to model the problem using graphs. The culprit's identity is to be revealed through a series of simple reasoning on interval graphs. The whole activity displays the main steps of research thinking in CS, namely concept definition, property analysis, modeling and mathematical reasoning. We discuss in detail the activity and its realization in outreach events for middle- and high-schoolers.

Keywords: CS Unplugged · Interval Graphs · Scientific Outreach · Research Thinking

1 Introduction

Computer science unplugged (CS Unplugged) has been, in recent years, a growing part of Computer Science related scientific outreach. It was introduced by Bell, Witten and Fellows [2], and follows the principle of introducing concepts of Computer Science without using computers or programming languages. It typically aims at popularizing a wide range of concepts occurring for example in cybersecurity, algorithmics or even machine learning. Yet, a significantly smaller portion of CS Unplugged work has been dedicated to so-called research thinking. Research thinking can be defined as the set of classical mental processes and

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methodology used in order to solve problems and further develop the knowledge in a specific research field. See [1] for a broader review of CS Unplugged and related popularization projects.

At a time when scientific expertise is questioned in public debates, helping students understand how scientific knowledge is constructed, *i.e.* through scientific thinking, has become a major educational challenge. Scientific knowledge increasingly shapes contemporary societies, and understanding how such knowledge is produced is arguably as important as learning its outcomes. Yet students in middle- and high-school rarely encounter the processes that characterize scientific work: exploring unfamiliar situations, formulating conjectures, identifying relevant evidence, revising ideas, and building convincing arguments. Scientific results often appear as finished products rather than as the outcome of a collective process of inquiry. Providing students with opportunities to experience these practices can help demystify scientific research. Computer Science, through carefully designed problems, offers a particularly fruitful setting in which students can engage in forms of reasoning closely related to those used by researchers. In particular, graph theory has been used to train and teach, through research-based methods, similar thought processes [5].

Graph theory allows to model a variety of problems and networks. Graph algorithms, in particular, have been designed for a range of everyday applications, such as shortest path computation, scheduling, or network monitoring. Several activities of CS Unplugged explore the popularization of Graph Theory through so called "pen-and-paper" activities[3], where the audience is introduced to some algorithms or some definitions on graphs with exclusively on-paper exercises or puzzles.

In this work, we propose a pen-and-paper scientific outreach activity designed for teenagers (in middle-school or high-school) whose goal is to popularize the thinking process of researchers in Graph Theory. To do so, we take inspiration from a novel published in 1994 by Claude Berge named *Who Killed the Duke of Densmore?* [3]. In this novel, a duke was assassinated and the detectives only manage to obtain from the suspects partial information about the murder. Suspects only disclosed who they had seen on the crime scene on the day the murder was committed. Fortunately, this information is sufficient to find out which suspect is guilty, thanks to properties of interval graphs. We adapt the previous story to get an activity where, after presenting the background story of a murder mystery, we define the mathematical objects that will be used to solve the mystery, namely interval graphs and their segment-based representation. A small section of exercises is proposed so that the participants can manipulate the definitions and understand better how both the graph-based and the segment-based representation are equivalent. Finally, the murder is solved through identifying forbidden structures on a graph modeling the information gathered by the de-

[3] See https://classic.csunplugged.org/activities/ including activites around search in graphs, the Dominating Set and Graph Colouring problems. See also https://sancy.iut.uca.fr/ iso/activites/equivalencegraphes/ where the concept of equivalence of problems on graphs is popularized (in French).

tectives. The conclusion of the activity highlights the steps of reasoning of the activity, and bring them in parallel with the research thinking performed by graph theory researchers.

Note that the graph used to solve the murder is exactly the one present in the original story by Berge, and has been kept as-is for this work. We designed, as an additional material, the exercises present in the Appendix, and integrated the story into a popularization of research thinking.

Some work has been dedicated to the popularization of research processes to a younger audience using discrete mathematics problems. Ouvrier-Buffet presents in [10] an activity based on polyominoes to introduce the concept of mathematical proofs and to have elementary pupils to experiment a simple research process. Modeling and proofs were also popularized in anterior work using discrete mathematics, and in particular graph theory [7].

The paper is organized as follows: Section 2 presents some scientific context around interval graphs and states the main results used during the activity, as well as related work concerning CS Unplugged outreach. Then, a thorough description of the activity is proposed in Section 3. Finally, we report in Section 4 how the activity was presented to teenagers during a national scientific outreach day in France. An Appendix containing a worksheet that was handed out to the participants is also included, for completeness.

2 Preliminaries

In this section, we present some graph notations and some properties of the interval graphs that will be used throughout the activity. Please note that understanding these notions does not require any specific technical background. A *graph* is a pair $G = (V, E)$ where $V = \{v_1, v_2, \ldots, v_n\}$ is a set of n *vertices* and $E \subseteq V \times V$ a set of m *edges*. An *interval graph* is a graph which can be represented as a *set of intervals* on the *real line*. More specifically, each interval corresponds to a vertex and an edge appears when two intervals intersect on the real line. In conclusion, interval graphs have two representations: a *graph-based* and a *segment-based* representation. See Figure 1 for examples of interval graphs and one of their segment-based representation.

Note that, for a single interval graph, many segment-based representations are valid.

Interval graphs are a broadly studied graph class due to its theoretical and real life applications. Several characterizations of interval graphs can be found in the literature [8]. Recognizing whether a given graph is an interval graph is tractable [4], and many problems that are computationally hard in general are tractable on interval graphs [6,9]. Applications using interval graphs range from scheduling problems, as intervals can easily represent time, to computational biology and network overseeing [9].

The key property of interval graphs that will be of main interest for the activity is the fact that some graphs do not admit a segment-based representation, *i.e.* they are not interval graphs. The simplest example of such a graph is the

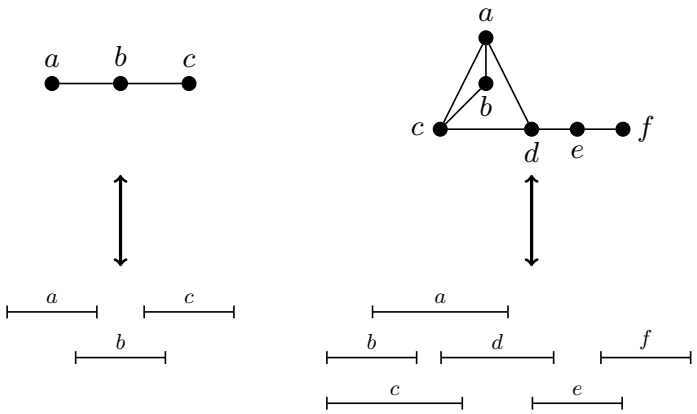


Fig. 1. Examples of interval graphs and one of their segment-based representation.

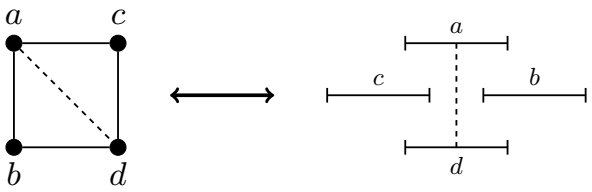


Fig. 2. The C_4 does not admit any segment-based representation and hence is not an interval graph. The dashed edge on the left part is required for the left segment-based representation to be valid.

(induced) cycle on four vertices, also denoted by C_4. Figure 2 illustrates the fact that no segment-based representation can represent a C_4.

Finally, we describe a simple property that will be used (and explained informally) during the activity in order to solve the murder case. For a set S of vertices of a graph $G = (V, E)$, we say that the subgraph of G *induced* by S, and denote by $G[S]$, the graph on vertices of S where the edges of $G[S]$ are exactly the edges of G whose both endpoints are in S. We say that a graph H is an *induced subgraph* of $G = (V, E)$ if there exists a set $S \subseteq V$ such that $H = G[S]$. It is easy to see that if a graph G is an interval graph, removing a vertex of G along with its adjacent edges will yield an interval graph. Thus, if some induced subgraph of G cannot be represented using segments, then G is not an interval graph. In particular, if a graph contains C_4 as an induced subgraph, then it is not an interval graph.

We also give another forbidden structure of interval graphs. The *neighborhood* a vertex $v \in V$ in graph $G = (V, E)$ is the set of vertices that share an edge of E with v. A *path* in G is a sequence of vertices of V such that any two neighboring vertices of the sequence share an edge of E. An *Asteroidal Triple* (AT) is a set S of three vertices such that there exists a path between each pair of S that

avoids the neighborhood of the third vertex of S. Such a structure is known to be forbidden in interval graphs [8].

3 Activity description

In this section, we describe the main features of the activity and precise some important steps of the presentation.

Introduction For the remainder, we suppose that the participants have no prior knowledge in graph theory. The first step of the activity aims at explaining the context of the murder story. At this step, we do not disclose any information about the testimonies that will be used in order to solve the murder. We only present the context and state the apparent main goal of the activity: finding the murderer. We will explicit links with research in Graph Theory at the end of the activity, as a conclusion. We also state at this point that we will introduce the audience to a new mathematical tool, namely graphs, in order to solve the murder.

Graphs and properties To introduce the concept of graph to the participants, we stress the need for some *modeling* of the situation, in order to easily express all the information gathered during the investigation. At this stage, we define the notion of graphs. Immediately, we exemplify this notion to the participants by using analogies with real-life. More specifically, we explain how graphs can model concepts or objects (such as human, house,...) with vertices, as well as relations between those concepts or objects (such as friendship, reachability,...) with edges. Afterwards, we introduce the concept of interval graph and their segment-based representation and have the participants practice their understanding of definitions with some exercises. They are required to find a segment-based representation of an interval graph and draw an interval graph based on its segment-based representation. Those exercises can be found in the Appendix. Finally, we briefly highlight that there exists some graphs that are not interval graph, taking C_4 as an example (see Figure 2) that we introduce as a *forbidden structure*. We try to draw a segment-based representation of C_4 one interval at a time, and convince the participants that no representation using segments is possible as we try to draw correctly the fourth segment.

Murder case description We recall the context and give more details to the participants about the murder case. To do so, we clearly state the problem faced (note that modifying the context of the story in order to adapt it to the public targeted is recommended in order to further encourage active participation of your audience): someone has been found dead and the police has been able to identify 8 suspects. The detectives were also able to get some information from each of the suspects: they all claim that they came only once during the murder day on the crime scene, and each suspect gave the list of the other suspects they saw on the crime scene. Moreover, due to the way the victim died, we know that

	Alice	Bob	Carole	Donald	Eve	Farid	Gontrand	Héloïse
Alice		x	x		x	x	x	
Bob	x		x					x
Carole	x	x		x	x			x
Donald			x		x			
Eve	x		x	x		x		
Farid	x				x			
Gontrand	x							x
Héloïse		x	x				x	

Table 1. Table of information used to construct the main graph of the activity.

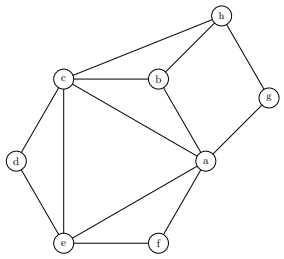


Fig. 3. The graph corresponding to the testimonies

the murderer must have come at least twice at the place to prepare the crime. For an immersive story, feel free to imagine why the murderer came at least twice (in the case of our story, the murderer came three times) such as hiding a bomb, prepare a bait for the victim, and finally detonate it.

Testimonies and modeling We then give the precise testimony of each of the suspects to the public, and have them compile and summarize it into a table (see Table 1). Once the table is filled, we check quickly, together with all the participants, that no two testimonies are contradictory by checking whether the table is symmetric. We then highlight that if each suspect only came once on the crime scene, their stay on the crime scene can be drawn an interval from the beginning and the end of their stay on some timeline. It means that we should be able to draw an interval graph based on the testimonies such that each suspect is represented by a unique interval. With this information in mind, we have the participants draw the graph whose adjacency matrix is the table filled previously. Each suspect is a vertex and there is an edge between two suspects if they have seen each other on the crime scene. The resulting graph is represented in Figure 3.

Solve the case Now, we let the audience find by themselves the forbidden structures of the graph, and recall that we presented in the first part of the presentation one forbidden structure. This allows the participants to find two induced C_4 in the graph, namely $[a, b, h, g]$ and $[a, c, h, g]$. Each forbidden structure found in the graph is some piece of evidence reducing the number of suspects. Indeed, if

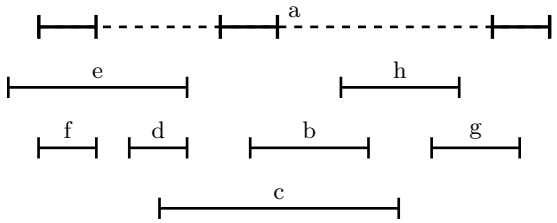


Fig. 4. A segment-based representation of the murder timeline. Alice had to come, in fact, three times for the testimonies to be coherent.

we find a forbidden structure, we know that one of the suspects associated to vertices of the forbidden structure must have lied. After finding both induced C_4 of the graph, the set of possible liars is restricted to three ($[a, h, g]$), but no other induced C_4 is present in the graph. A new structure has to be identified, and we give the participants, after a moment, the structure to be found and sketch a proof that it cannot be represented with segments, which is enough to designate the culprit which is in fact Alice. In fact, this structure is $G[\{a, b, c, d, e, f\}]$. This subgraph is a forbidden structure because it contains an AT. Indeed, it can be verified that the set $\{b, d, f\}$ is an AT. For the sake of simplicity, we do not introduce this notion to the participants, and we try to draw, as previously done for the C_4, a valid interval representation of $G[\{a, b, c, d, e, f\}]$. First, we notice that $[a, c, e]$ are all connected together, and draw then three lines of the same length each above the others. Then, to add b, we extend a and c a bit to the left such that b does not cross e. We do the same for d to the right with c and e. Finally, we ask the participant how they would add f from now on, and, most of the time, a participant says that it is not possible. It remains to use this new evidence to reduce the amount of suspects. Indeed, only Alice is in the three identified forbidden structures, and is thus guilty. Depending on the time left for the activity, it is possible but not mandatory to show how the actual, valid interval representation of the situation. In reality, Alice came three times to the crime scene (see Figure 4).

Conclusion We end the activity by giving a brief summary of what the participants have learned, and what we could achieve using the concept of interval graph. As a final word, we explicit further how to map the steps of the activity with research.

- We started from a specific problem and look for some model that could represent it, here graphs.
- We defined some concepts and prove results or properties on those. Here, we defined the class of interval graphs and showed that some graphs were not a part of this class. We also stress at this point that this class of graphs is used in real life applications, as stated in the introduction of this paper.
- Then, using the model associated to the original problem, we performed some reasoning in order to obtain some information on the problem, and

eventually solve it. Here, we spotted some forbidden structures that helped reducing the list of suspects.

We end the session by explaining that this thought process is central in scientific thinking and that the participants, for the time of the presentation, were put in a "guided" research situation and applied very similar thought process to the ones used by researchers.

4 Activity report

Every year, a national scientific outreach event called *La Fête de la Science* ("The Science Festival") is organized in France in order to promote science and encourage scientists to share their knowledge with the general public. We presented the activity described in this paper to middle-schoolers and high-schoolers during this event in Clermont-Ferrand, France. Around ten groups of about 15 students participated throughout the day, and we spent with each group around 45 minutes. Those students have *a priori* no knowledge of graph theory before the activity, and usually do not have a substantial prior experience with algorithms. The students were accompanied by teachers (usually not mathematics teachers) and we had them also participating in the activity. The activity was well received by a majority of the participants. Thanks to the early introduction of the challenge of the activity (namely finding the culprit), the students were curious and eager to solve the mystery. No particular misconception by the participants were noticed, but for a few groups, the link between a forbidden structure and how to reduce the number of potential culprits had to be explained twice in order for the audience to fully understand this link.

The small exercise session after introducing the different notions in the middle of the activity seemed to help the students to understand the presented notions, since each group quite easily found the first two forbidden structures in the graph of suspects. One hint that had to be given to almost all groups during the exercise session was that edges do not need to be drawn as straight lines, as this (intentionally) never was stated when presenting graphs. This was another way to put participants in a situation where they had to question definitions and assumptions, as a researcher could. All groups could solve the different exercises in the time limit we imposed (around 10 minutes).

Only one group struggled to find a second C_4. Interestingly, some groups also found without major hints that the C4 structure was not sufficient in order to discriminate between the two last suspects. These groups proposed spontaneously some other structures (or at least expressed that there was no more induced C_4 in the graph), and for one of the groups, the proposition was in fact the correct one.

The majority of the teachers mentioned that the presentation was clear and helpful to demystify research in graph theory and more broadly in Theoretical Computer Science. They specifically enjoyed the progression of the activity, especially the conclusion part that they did not anticipated, as the real goal of the activity was not disclosed before the conclusion.

5 Conclusion

We presented an outreach activity that illustrates, through solving a toy murder case in a CS Unplugged fashion, some core elements of scientific thinking process in Theoretical Computer Science. Further work might naturally try to extend the set of activities aiming at further popularizing research thinking to middle- and high-schoolers.

Acknowledgment

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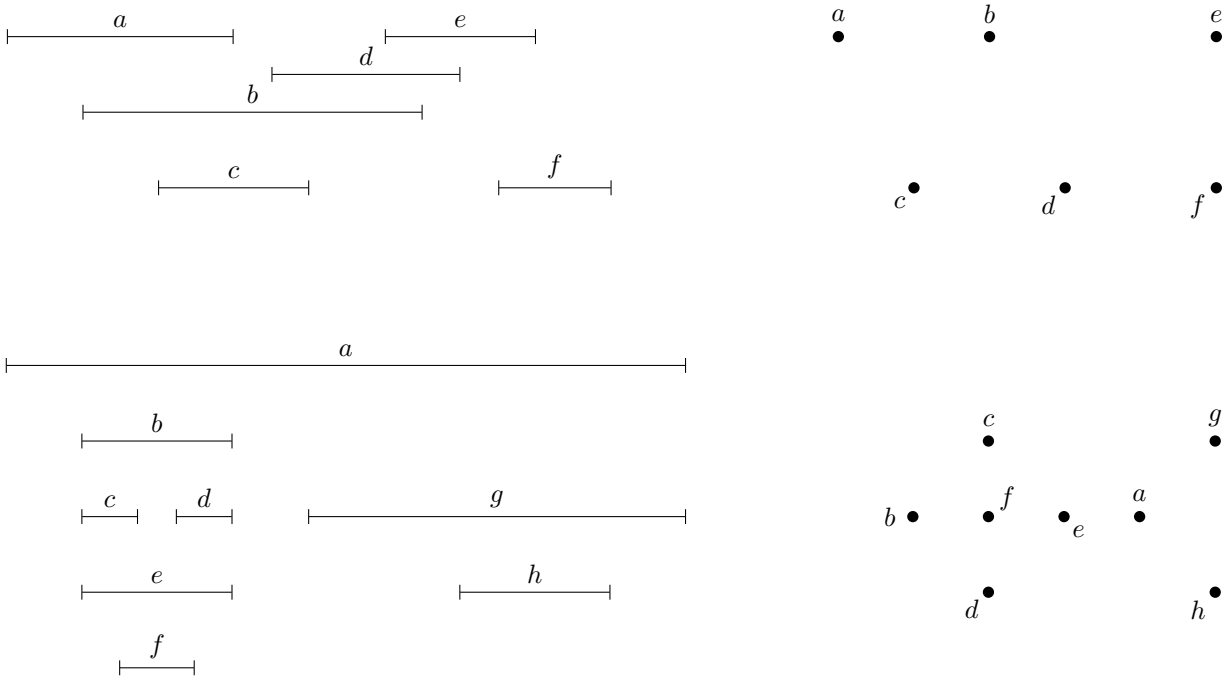
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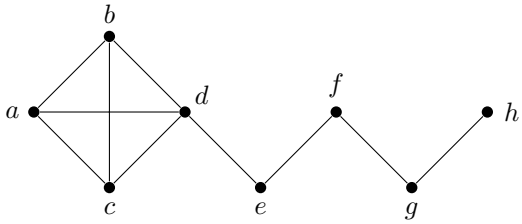
A Exercise sheet

The material on the next two pages was handed out to the participants in order to have them interact with the notions presented. It contains three small exercises on interval graphs and their representations, as well as some table to help gathering the pieces of information of the investigation.

Draw the corresponding interval graphs



Draw a segment-based representation corresponding to this graph



Let's solve the mystery!

	Alice	Bob	Carole	Donald	Eve	Farid	Gontrand	Héloïse
Alice								
Bob								
Carole								
Donald								
Eve								
Farid								
Gontrand								
Héloïse								

