Vérification de protocoles cryptographiques en présence de théories équationnelles

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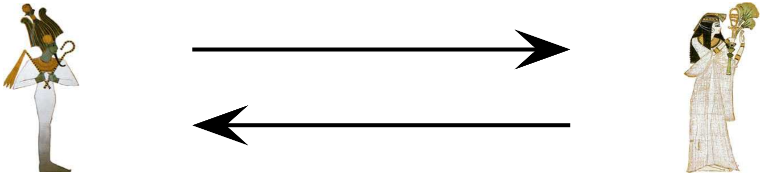
Cachan : September 25th 2006

Cryptographic Protocols



Osiris communicates with Isis via the net.

Cryptographic Protocols

Intruder

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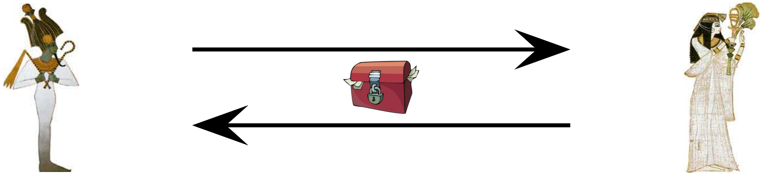
Cryptographic Protocols

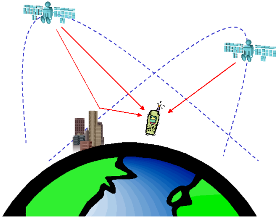


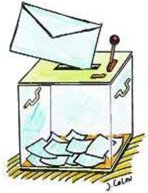
Intruder

Osiris communicates with Isis via the net.

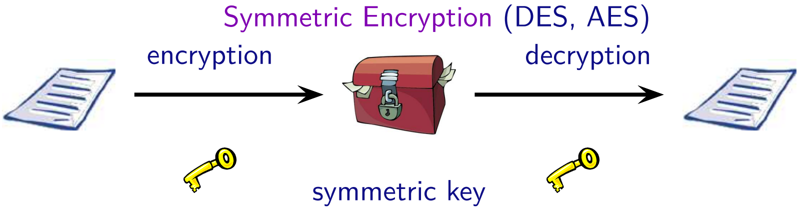
Secrecy Property: Intruder cannot learn a *secret data.*

Applications





Cryptography



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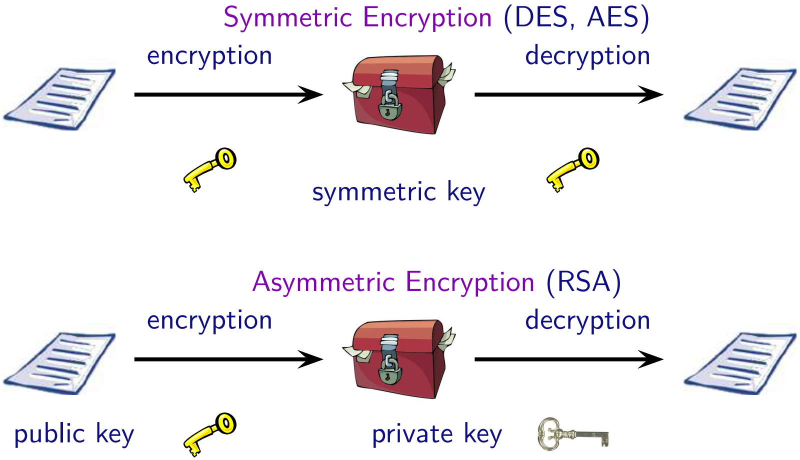

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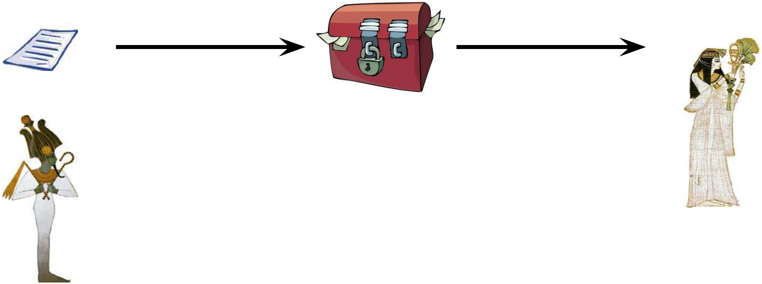

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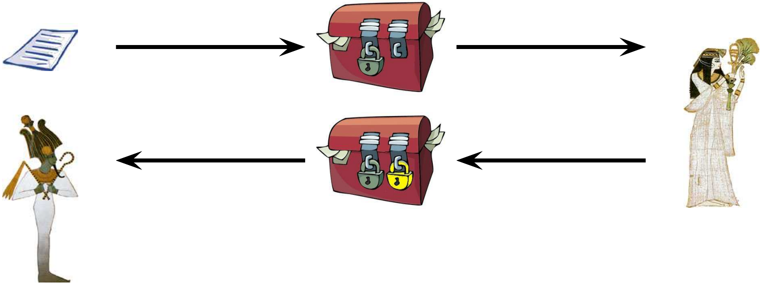

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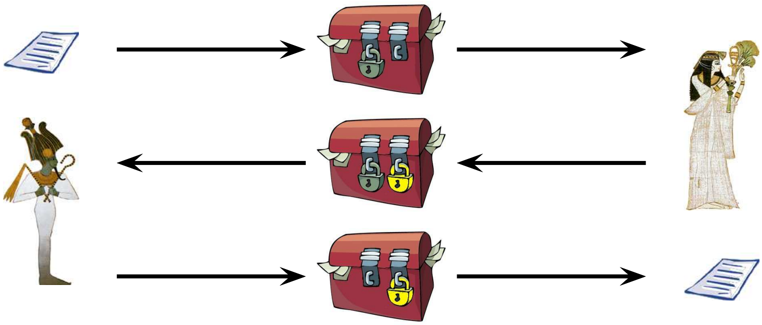

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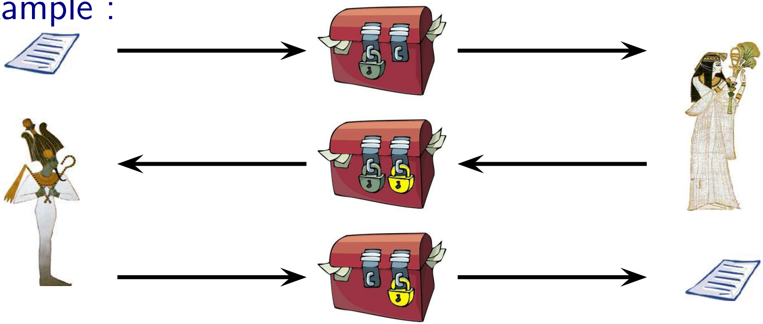
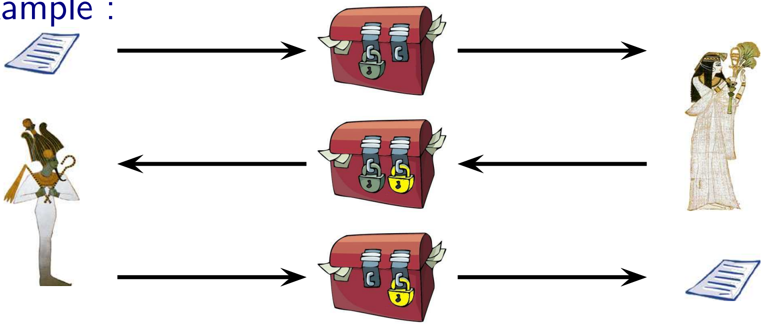


Shamir 3-Pass Protocol

$$1 \quad O \quad \rightarrow \quad I \quad : \quad \{m\}_{K_O}$$

Example :



Shamir 3-Pass Protocol

$$\begin{array}{llcll} 1 & O & \to & I & : \{m\}_{K_O} \\ 2 & I & \to & O & : \{\{m\}_{K_O}\}_{K_I} \end{array}$$

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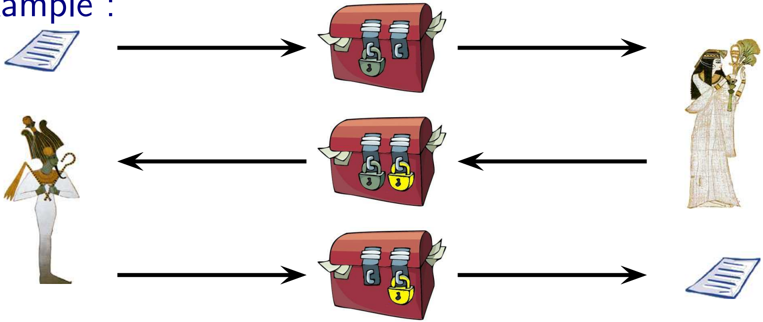

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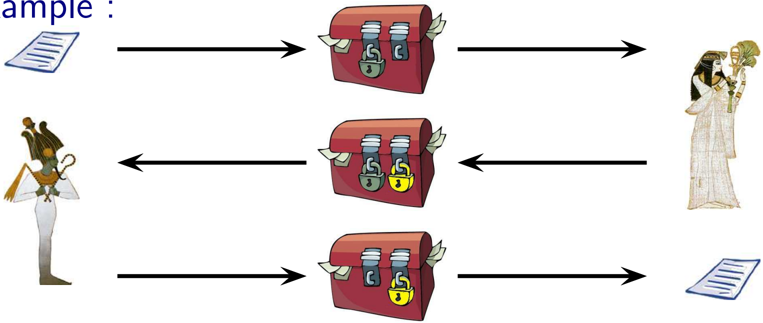
Commutative Encryption

Example :



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Commutative Encryption

Attacks

Cryptanalysis

Attacks

Cryptanalysis





Attacks

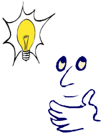
Cryptanalysis

Logical Attack

Perfect Encryption hypothesis

Needham-Schroeder Public Key Protocol (1978)

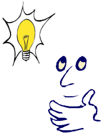
“Man in the middle attack” [Lowe’96]



Attacks

Cryptanalysis

Logical Attack + Algebraic properties



Perfect Encryption hypothesis

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Formal Approach

Symbolic abstraction

- Messages represented by terms
  - $\{m\}_k$
  - $\langle m_1, m_2 \rangle$
- Perfect encryption hypothesis

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Useful abstraction [Clark & Jacob'97]

Automatic verification with Tools:
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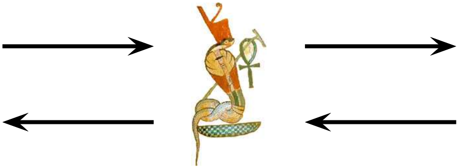

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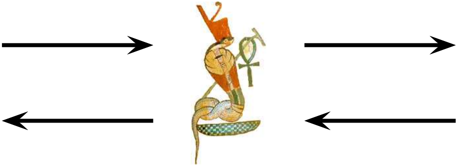

Listen

Passive: Intruder deduction problem

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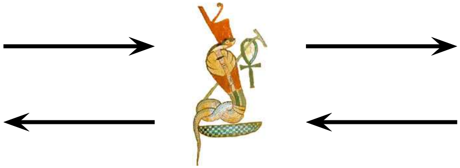

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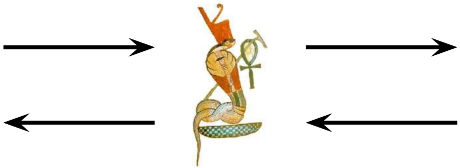

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- Pairing, Projection.

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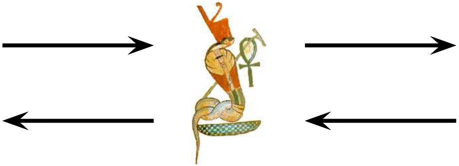

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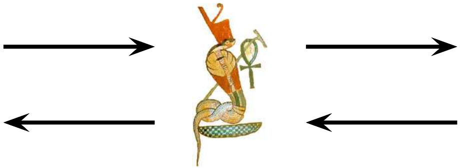

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Bounded number of session $\Rightarrow$ Decidability [AL'00, RT'01]

Logical Attack on Shamir 3-Pass Protocol (I)

Perfect encryption one-time pad (Vernam Encryption)

$\{m\}_k = m \oplus k$

XOR Properties (ACUN)

- $(x \oplus y) \oplus z = x \oplus (y \oplus z)$ **A**ssociativity
- $x \oplus y = y \oplus x$ **C**ommutativity
- $x \oplus 0 = x$ **U**nity
- $x \oplus x = 0$ **N**ilpotency

Logical Attack on Shamir 3-Pass Protocol (I)

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Vernam encryption is a commutative encryption :

$$\{\{m\}_{K_O}\}_{K_I} = (m \oplus K_O) \oplus K_I = (m \oplus K_I) \oplus K_O = \{\{m\}_{K_I}\}_{K_O}$$

Logical Attack on Shamir 3-Pass Protocol (II)

Perfect encryption one-time pad (Vernam Encryption)

$\{m\}_k = m \oplus k$

Shamir 3-Pass Protocol

$$\begin{array}{lllll} 1 & O & \rightarrow & I: & m \oplus K_O \\ 2 & I & \rightarrow & O: & (m \oplus K_O) \oplus K_I \\ 3 & O & \rightarrow & I: & m \oplus K_I \end{array}$$

Passive attacker :

$$m \oplus K_O \qquad m \oplus K_O \oplus K_I \qquad m \oplus K_I$$

Logical Attack on Shamir 3-Pass Protocol (II)

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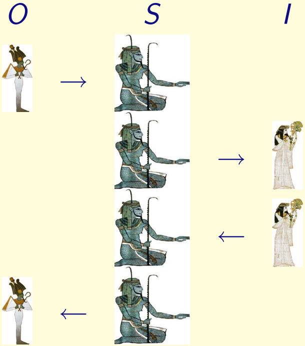
Passive attacker :

$$m \oplus K_O \oplus m \oplus K_O \oplus K_I \oplus m \oplus K_I = m$$

TMN Protocol: Distribution of a fresh symmetric key

[Tatebayashi, Matsuzuki, Newmann 89]:



$O \rightarrow S$: $O, I, \{N_O\}_{Pub_S}$

$S \rightarrow I$: S, O

$S \leftarrow I$: $I, O, \{N_I\}_{Pub_S}$

$O \leftarrow S$: $S, I, N_O \oplus N_I$

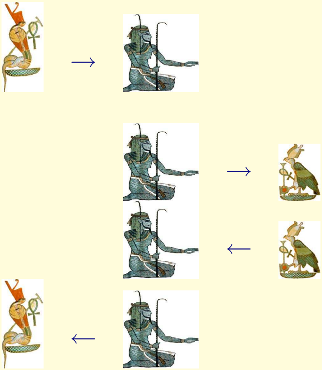
Osiris retrieves N_I:

Using $x \oplus x \oplus y = y$, knowing N_O

Attack on TMN Protocol [Simmons'94]

With homomorphic encryption $\{a\}_k \oplus \{b\}_k = \{a \oplus b\}_k$

B $\quad$ S $\quad$ C



$B \rightarrow S$: $B, C, \underbrace{\{N_I\}_{Pub_S} \oplus \{N_B\}_{Pub_S}}_{\{N_I \oplus N_B\}_{Pub_S}}$

$S \rightarrow C$: S, B

$S \leftarrow C$: $C, B, \{N_C\}_{Pub_S}$

$B \leftarrow S$: $S, (N_I \oplus N_B) \oplus N_C$

Buto Learns:

Using $x \oplus x \oplus y = y$, knowing N_B and N_C, he deduces N_I.

Relaxing the perfect encryption hypothesis.

[Journal of Computer Security'06]

	Examples of Protocols	Intruder Deduction Problem	Security Problem
Commutative encryption	Shamir	*P-TIME [CKRT'04]*	*NP-Complete [CKRT'04]*
ACUN	Bull, Gong	*P-TIME [CS'03,CKRT'03]*	*NP-Complete [CS03,CKRT'03]*
AG + Exp	IKA.1	*P-TIME [CKRTV'03]*	*Decidable [MS'03]*
ACUNh	WEP	*?*	*?*
AGh	TMN	*?*	*?*

Combination Result [CR'06]

My contributions : Homomorphism property

Passive Intruder

- $h(x \oplus y) = h(x) \oplus h(y)$ [RTA'05]
  - ACh
  - ACUNh
  - AGh
- $\{x \oplus y\}_k = \{x\}_k \oplus \{y\}_k$
  - Distributive Encryption [I&C] Submitted (ACUN$\{.\}_.$, AG$\{.\}_.$)
  - Distributive and Commutative Encryption [Secret'06] $\{\{m\}_{k_1}\}_{k_2} = \{\{m\}_{k_2}\}_{k_1}$

Active Intruder [ICALP'05]

- ACUNh

Outline

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Extended Dolev-Yao Deduction System

Deduction System : $T_0 \vdash^? s$

(A) $\dfrac{u \in T_0}{T_0 \vdash u}$

(UL) $\dfrac{T_0 \vdash \langle u, v\rangle}{T_0 \vdash u}$

(P) $\dfrac{T_0 \vdash u \qquad T_0 \vdash v}{T_0 \vdash \langle u, v\rangle}$

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(F) $\dfrac{T_0 \vdash u_1 \quad \cdots \quad T_0 \vdash u_n}{T_0 \vdash f(u_1, \ldots, u_n)}$

$$(Eq)\dfrac{T_0 \vdash u \qquad u =_E v}{T_0 \vdash v}$$

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E is represented by a confluent and terminating rewriting system modulo AC ↓

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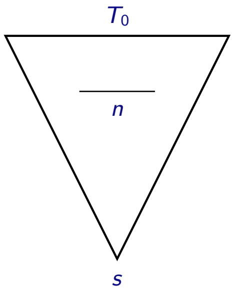
E is represented by a confluent and terminating rewriting system modulo AC $\downarrow$

Definition of S-Locality

- A proof P of $T_0 \vdash s$ is S-local :

$$\forall n \in P, n \in S(T_0 \cup \{s\})$$



Extended McAllester's Theorem

Let P be a proof system, if:

- P is S-local,
- the size of $S(T)$ is computable with complexity K_2,
- one-step deducibility is decidable with complexity K_1,

then provability in the proof system P is decidable in $max(K_1, K_2)$.

Intruder Deduction problem [RTA'05]

Dolev-Yao Model extended by:

$$(GX)\frac{T_0 \vdash_E u_1 \qquad \ldots \qquad T_0 \vdash_E u_n}{T_0 \vdash_E (u_1 \oplus \ldots \oplus u_n)\downarrow} \quad (h)\frac{T_0 \vdash_E u}{T_0 \vdash_E h(u)\downarrow}$$

NP-Complete ACh

- One-step deducibility ($\mathbb{N}$)
- Syntactic Subterms (P-TIME) and locality: easy

EXP-TIME ACUNh, AGh

- One-step deducibility: easy ($\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}$)
- Design S (EXP-TIME) and prove locality: difficult

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P-TIME Complete [Delaune'06] ACUNh, AGh

Distributive Encryption : ACUN$\{.\}_.$, AG$\{.\}_.$

$$\{x \oplus y\}_k = \{x\}_k \oplus \{y\}_k$$

Example

$T_0 = \{a \oplus \{b\}_k, \{b\}_k \oplus c, \{c\}_k \oplus d, k\}$ and $s = \{a\}_k \oplus d$
$S(T_0 \cup \{s\}) = T_0 \cup \{s, a, b, c, d, k, \{a\}_k, \{b\}_k, \{c\}_k\}$

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$$\Downarrow$$

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Intruder Deduction Problem

ACUN$\{.\}_.$

- Atomic Locality Result
- EXP-TIME decision procedure

AG$\{.\}_.$

- Atomic Locality Result & $\mathbb{Z}$-module
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Binary Case

$\forall n \in P, n = .$ or $n = . \oplus .$ but $\cancel{n = . \oplus . \oplus \ldots \oplus .}$

- AG$\{.\}_.$ P-TIME (prefix rewriting)

Intruder Deduction Problem

ACUN$\{.\}_.$ and Commutative Encryption

- Atomic Locality Result
- 2-EXP-TIME decision procedure

AG$\{.\}_.$ and Commutative Encryption

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(D) $$\frac{T_0 \vdash \{u\}_v \qquad T_0 \vdash v}{T_0 \vdash u}$$

(M_E) $$\frac{T_0 \vdash u_1 \quad \cdots \quad T_0 \vdash u_n}{T_0 \vdash C[u_1, \ldots, u_n]}$$ C is a context made with $\{h, \oplus\}$

New Extended Dolev-Yao Deduction System

Deduction System : $T_0 \vdash^? s$

(A) $\dfrac{u \in T_0}{T_0 \vdash u}$

(UL) $\dfrac{T_0 \vdash \langle u, v \rangle}{T_0 \vdash u}$

(P) $\dfrac{T_0 \vdash u \qquad T_0 \vdash v}{T_0 \vdash \langle u, v \rangle}$

(UR) $\dfrac{T_0 \vdash \langle u, v \rangle}{T_0 \vdash v}$

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(M_E) $\dfrac{T_0 \vdash u_1 \quad \cdots \quad T_0 \vdash u_n}{T_0 \vdash C[u_1, \ldots, u_n]}$ C is a context made with $\{h, \oplus\}$

Example

$$\dfrac{T_0 \vdash a \oplus h(a) \quad T_0 \vdash b}{T_0 \vdash a \oplus h^2(a) \oplus h(b)}$$

$C[u_1, u_2] = (h \oplus 1)(u_1) \oplus h(u_2)$

$a \oplus h(a) \quad \oplus \quad h(a \oplus h(a)) \quad \oplus \quad h(b)$

Modeling a protocol as a system of constraints

The Intruder is the network, he can listen, build, send and replay messages.

$$\mathcal{P} \quad := \quad \begin{cases} recv(u_1); send(v_1) \\ recv(u_2); send(v_2) \\ \quad\quad\quad \vdots \\ recv(u_n); send(v_n) \end{cases}$$

T_0 initial Intruder knowledge.

$$\mathcal{C} \quad := \quad \begin{cases} T_0 & \Vdash & u_1 \\ T_0, v_1 & \Vdash & u_2 \\ T_0, v_1, v_2 & \Vdash & u_3 \\ & \vdots & \\ T_0, v_1, \ldots, v_n & \Vdash & s \end{cases}$$

If this system has a solution σ then the secret s can be obtained by the Intruder.

System of Constraints Well-formed [MS'03]

$\mathcal{C} = \{T_i \Vdash u_i\}_{1 \le i \le k}$ is well-formed if:

- monotonicity: The knowledge of the intruder is increasing.

$$T_1 \subseteq T_2 \subseteq \ldots \subseteq T_k$$

- origination: Variables appear first on right side:

$$x \in vars(T_i) \Rightarrow \exists j < i \text{ such that } : x \in vars(u_j)$$

System of Constraints Well-defined [MS'03]

$\mathcal{C}$ is well-defined if for every substitution θ, $\mathcal{C}\theta \downarrow$ is well-formed.

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Well-Definedness: Example

$$\mathcal{C} \quad := \quad \begin{cases} T_0 & \Vdash \quad X \oplus Y \\ T_0, X & \Vdash \quad c \end{cases}$$

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Monotonicity OK !

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$$\mathcal{C} \quad := \quad \left\{ \begin{array}{lcl} T_0 & \Vdash & X \oplus Y \\ T_0, X & \Vdash & c \end{array} \right.$$

Monotonicity OK !

Origination OK !

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Monotonicity OK !

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Well-formed OK !

Well-Definedness: Example

$$\mathcal{C} \quad := \quad \left\{ \begin{array}{lll} T_0 & \Vdash & X \oplus Y \\ T_0, X & \Vdash & c \end{array} \right.$$

Monotonicity OK !

Origination OK !

Well-formed OK !

But NOT well-defined !

$\theta = \{Y \to X\}$ and $\mathcal{C}\theta$ is not well-formed:

$$\mathcal{C}\theta \quad := \quad \left\{ \begin{array}{lll} T_0 & \Vdash & 0 \\ T_0, X & \Vdash & c \end{array} \right.$$

Our Procedure

Theorem [ICALP'06]

The security problem modulo ACUNh with a bounded number of sessions is decidable for deterministic protocols.

Idea of the proof:
Let $\mathcal{C}$ be a W-D constraints system

1. From W-D $\Vdash$ to W-D $\Vdash_1$
2. From W-D $\Vdash_1$ to W-D $\Vdash_{M_E}$
3. From W-D $\Vdash_{M_E}$ to W-D equations systems
4. Solve these W-D equations systems

From W-D $\Vdash$ to W-D $\Vdash_1$

Example

$$\mathcal{C} \quad := \quad T \quad \Vdash \quad \langle X, h(Y) \rangle$$

Guess set of subterms of $\mathcal{C}$ and an order on these subterms

$$\mathcal{C}' \quad := \quad \begin{cases} T & \Vdash_1 \quad X \\ T, X & \Vdash_1 \quad h(Y) \\ T, X, h(Y) & \Vdash_1 \quad \langle X, h(Y) \rangle \end{cases}$$

From W-D $\Vdash$ to W-D $\Vdash_1$

Example

$$\mathcal{C} \quad := \quad T \quad \Vdash \quad \langle X, h(Y) \rangle$$

Guess set of subterms of $\mathcal{C}$ and an order on these subterms

$$\mathcal{C}' \quad := \quad \begin{cases} T & \Vdash_1 \quad X \\ T, X & \Vdash_1 \quad h(Y) \\ T, X, h(Y) & \Vdash_1 \quad \langle X, h(Y) \rangle \end{cases}$$

From W-D $\Vdash_1$ to W-D $\Vdash_{M_E}$

Guess equalities between subterms of $\mathcal{C}$.
(consider all the possible applications of rules (C) (P) (D) (UR) (UL))

Example

$$\mathcal{C} \quad := \quad \begin{cases} \langle a, b\rangle & \Vdash_1 \quad \langle X, b\rangle \\ \langle a, b\rangle, X \oplus b & \Vdash_1 \quad Y \oplus \langle a, b\rangle \end{cases}$$

Guess $\{\langle X, b\rangle = \langle a, b\rangle\}$, compute ACUNh m.g.u. $\theta : \{X \mapsto a\}$ [UNIF'06]

$$\mathcal{C}\theta \quad := \quad \begin{cases} \langle a, b\rangle & \Vdash_{M_E} \quad \langle a, b\rangle \\ \langle a, b\rangle, a \oplus b & \Vdash_{M_E} \quad Y \oplus \langle a, b\rangle \end{cases}$$

From W-D $\Vdash_{M_E}$ to W-D Equations System (I)

Idea

Abstraction ρ to get a constraint system on signature: $\oplus$, h, and constant symbols.

Example:

$$\mathcal{C} \quad := \quad \begin{cases} a, b & \Vdash_{M_E} \quad \langle X, b \rangle \\ a, b, X & \Vdash_{M_E} \quad X \oplus b \end{cases}$$

$\mathcal{C}$ is well-defined, but not $\mathcal{C}\rho$

$$\mathcal{C}\rho \quad := \quad \begin{cases} a, b & \Vdash_{M_E} \quad c_1 \\ a, b, X & \Vdash_{M_E} \quad X \oplus b \end{cases}$$

From W-D $\Vdash_{M_E}$ to W-D Equations System (II)

Lemma

Restriction to systems where abstraction preserves Well-Definedness is sufficent for completeness.

Example:

$$\mathcal{C} \quad := \quad \begin{cases} a, b & \Vdash_{M_E} \quad X \\ a, b, \langle X, b \rangle & \Vdash_{M_E} \quad \langle X, b \rangle \oplus Z \end{cases}$$

$\mathcal{C}$ and $\mathcal{C}\rho$ are well-defined.

$$\mathcal{C}\rho \quad := \quad \begin{cases} a, b & \Vdash_{M_E} \quad X \\ a, b, c_1 & \Vdash_{M_E} \quad c_1 \oplus Z \end{cases}$$

Constraint M_E to Quadratic Equations System

System $\mathcal{C}$ of Constraints M_E

$$\mathcal{C} \quad := \quad \begin{cases} t_1, t_2 & \Vdash_{\mathrm{M_E}} \quad h(X_1) \oplus X_2 \\ t_1, t_2, X_1 \oplus X_2 & \Vdash_{\mathrm{M_E}} \quad X_1 \oplus a \\ t_1, t_2, X_1 \oplus X_2, X_1 & \Vdash_{\mathrm{M_E}} \quad X_2 \oplus b \end{cases}$$

System of equations $\mathcal{E}$

$$\mathcal{E} \quad := \quad \begin{cases} z[1,1]t_1 \oplus z[1,2]t_2 & = \quad h(X_1) \oplus X_2 \\ z[2,1]t_1 \oplus z[2,2]t_2 \oplus z[2,3](X_1 \oplus X_2) & = \quad X_1 \oplus a \\ z[3,1]t_1 \oplus z[3,2]t_2 \oplus z[3,3](X_1 \oplus X_2) \oplus z[3,4]X_1 & = \quad X_2 \oplus b \end{cases}$$

$z[i,j] \in \mathbb{Z}/2\mathbb{Z}[h]$

Solving quadratic systems of equations is in general undecidable.

Constraint M_E to Quadratic Equations System

System $\mathcal{C}$ of Constraints M_E

$$\mathcal{C} \quad := \quad \left\{ \begin{array}{lll} t_1, t_2 & \Vdash_{\mathsf{M_E}} & h(X_1) \oplus X_2 \\ t_1, t_2, X_1 \oplus X_2 & \Vdash_{\mathsf{M_E}} & X_1 \oplus a \\ t_1, t_2, X_1 \oplus X_2, X_1 & \Vdash_{\mathsf{M_E}} & X_2 \oplus b \end{array} \right.$$

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$$\mathcal{E} \quad := \quad \left\{ \begin{array}{lll} z[1,1]t_1 \oplus z[1,2]t_2 & = & h(X_1) \oplus X_2 \\ z[2,1]t_1 \oplus z[2,2]t_2 \oplus z[2,3](X_1 \oplus X_2) & = & X_1 \oplus a \\ z[3,1]t_1 \oplus z[3,2]t_2 \oplus z[3,3](X_1 \oplus X_2) \oplus z[3,4]X_1 & = & X_2 \oplus b \end{array} \right.$$

$z[i,j] \in \mathbb{Z}/2\mathbb{Z}[h]$

Solving quadratic systems of equations is in general undecidable.

We propose a procedure to solve Well-defined Quadratic system of equations.

Outline

Theorem [ICALP'06]

The security problem modulo ACUNh with a bounded number of sessions is decidable for deterministic protocols.

Given: Well-defined protocol.

1. Guess partition of subterms $\Rightarrow$ WD one-step Constraints
2. Guess equality on subterms $\Rightarrow$ WD M_E Constraints
3. Abstraction $\Rightarrow$ System of equations WD
4. Solve system of equations $\Rightarrow$ Attack on Protocol.

Conclusions & Future Works

	Complexity	
	Intruder Deduction Problem	**Security Problem**
ACUNh	*EXP-TIME* [RTA'05]	*Decidable* [ICALP'06]
AGh	*EXP-TIME* [RTA'05]	*Undecidable*
ACUN$\{.\}_.$ **&** **AG**$\{.\}_.$	*EXP-TIME* Submitted	*?*
ACUN$\{.\}_.$ **&** **AG**$\{.\}_.$ **Commutative**	*2EXP-TIME* [Secret'06]	*?*

Future Works

Extensions

- Active case for distributive encryption.
- Complexity of our procedure (Active Case).
- General procedure for monoidal theories (AG,ACUN).

Applications

- Web Services.
- Elliptic curves.
- Others properties on new protocols: Authentification, Fairness, Timestamps...
  - E-auction
  - Wireless

Thank you for your attention.

Questions ?