Automatic Proofs for Symmetric Encryption Modes

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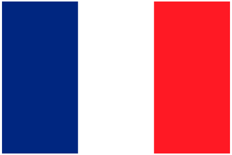
Indistinguishability and Symmetric Encryption Modes



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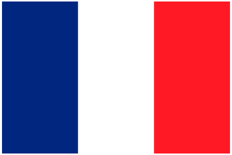

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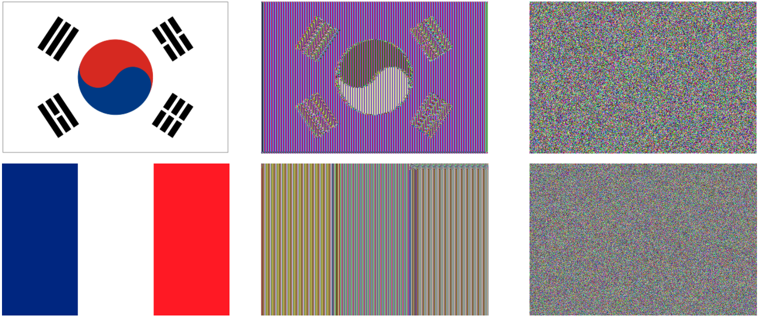


ECB

CBC, OFB ...

Block Cipher Modes

$$\text{PRP } \mathcal{E} \rightarrow \boxed{\text{Encryption Mode}} \rightarrow \text{ IND-CPA}$$

NIST standard

- Electronic Code Book (ECB)
- Cipher Block Chaining (CBC)
- Cipher FeedBack mode (CFB)
- Output FeedBack (OFB), and
- Counter mode (CTR).

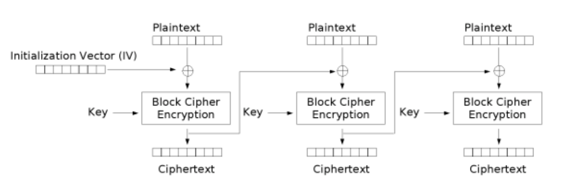
Others

DMC,CBC-MAC, IACBC, IAPM, XCB ,TMAC, HCTR, HCH, EME, EME*, PEP, OMAC, TET, CMC, GCM, EAX, XEX, TAE, TCH, TBC, CCM, ABL4

Block Cipher Modes

Example

Cipher Block Chaining (CBC)



Cipher Block Chaining (CBC) mode encryption

$$C_i = \mathcal{E}(P_i \oplus C_{i-1}), C_0 = IV$$

CBC and others

CBC	CTR	OFB	CFB
$IV \xleftarrow{\$} \mathcal{U};$	$IV \xleftarrow{\$} \mathcal{U};$	$IV \xleftarrow{\$} \mathcal{U};$	$IV \xleftarrow{\$} \mathcal{U};$
$z_1 := IV \oplus m_1;$	$z_1 := \mathcal{E}(IV + 1);$	$z_1 := \mathcal{E}(IV);$	$z_1 := \mathcal{E}(IV);$
$c_1 := \mathcal{E}(z_1);$	$c_1 := m_1 \oplus z_1;$	$c_1 := m_1 \oplus z_1;$	$c_1 := m_1 \oplus z_1;$
$z_2 := c_1 \oplus m_2;$	$z_2 := \mathcal{E}(IV + 2);$	$z_2 := \mathcal{E}(z_1);$	$z_2 := \mathcal{E}(c_1);$
$c_2 := \mathcal{E}(z_2);$	$c_2 := m_2 \oplus z_2;$	$c_2 := m_2 \oplus z_2;$	$c_2 := m_2 \oplus z_2;$
$z_3 := c_2 \oplus m_3;$	$z_3 := \mathcal{E}(IV + 3);$	$z_3 := \mathcal{E}(z_2);$	$z_3 := \mathcal{E}(c_2);$
$c_3 := \mathcal{E}(z_3);$	$c_3 := m_3 \oplus z_3;$	$c_3 := m_3 \oplus z_3;$	$c_3 := m_3 \oplus z_3;$

Outline

How to prove an encryption mode is IND-CPA ?

Our Approach

Automated method for proving correctness of encryption mode:

- Language: Generic Encryption Mode
- Predicates: F, E, Indis, Rcounter
- Hoare logic : 20 rules

RESULT:

If a Generic Encryption Mode $\mathcal{E}_M$ is correct according to our Hoare logic then $\mathcal{E}_M$ is IND-CPA.

Grammar

$$
\begin{aligned}
\mathsf{c} \quad ::= \quad & x \xleftarrow{\$} \mathcal{U} \mid x := \mathcal{E}(y) \mid x := y \oplus z \mid x := y \| z \mid \\
& x := y + 1 \mid c_1 ; c_2
\end{aligned}
$$

Generic Encryption Mode

Definition

A generic encryption mode M is represented by

$$\mathcal{E}_M(m_1|\dots|m_p, c_0|\dots|c_p) : \textbf{var } \vec{x}; \mathrm{c}$$

$$\begin{aligned}
&\mathcal{E}_{CBC}(m_1|m_2|m_3, IV|c_1|c_2|c_3) : \\
&\textbf{var } z_1, z_2, z_3; \\
&IV \xleftarrow{\$} \mathcal{U}; \\
&z_1 := IV \oplus m_1; \\
&c_1 := \mathcal{E}(z_1); \\
&z_2 := c_1 \oplus m_2; \\
&c_2 := \mathcal{E}(z_2); \\
&z_3 := c_2 \oplus m_3; \\
&c_3 := \mathcal{E}(z_3);
\end{aligned}$$

Predicates

$$\psi ::= \mathrm{Indis}(\nu x; V) \mid F(e) \mid \mathrm{E}(\mathcal{E}, e) \mid Rcounter(e)$$
$$\varphi ::= \mathrm{true} \mid \varphi \wedge \varphi \mid \psi,$$

Indis$(\nu x; V)$: The value of x is indistinguishable from a random value given the value of the variables in V.

$F(e)$: The value of e is indistinguishable from a random value that has not been used before.

$\mathrm{E}(\mathcal{E}, e)$: The probability that the value of e have been encrypted by $\mathcal{E}$ is negligible.

$RCounter(e)$: e is the most recent value of a monotone counter that started at a fresh random value.

How to generate $\mathsf{E}(\mathcal{E}, x)$?

Sampling a Random

(R1) $\{true\}\ x \stackrel{\$}{\leftarrow} \mathcal{U}\ \{F(x) \wedge \mathsf{Indis}(\nu x) \wedge \mathsf{E}(\mathcal{E}, x)\}$

How to generate $\mathsf{E}(\mathcal{E}, x)$?

Sampling a Random

(R1) $\{true\}\ x \xleftarrow{\$} \mathcal{U}\ \{F(x) \wedge \mathsf{Indis}(\nu x) \wedge \mathsf{E}(\mathcal{E}, x)\}$

PRP Encryption

(B1) $\{\mathsf{E}(\mathcal{E}, y)\}\ x := \mathcal{E}(y)\ \{F(x) \wedge \mathsf{Indis}(\nu x) \wedge \mathsf{E}(\mathcal{E}, x)\}$

How to generate $\mathsf{E}(\mathcal{E}, x)$?

Xor

(X4) $\{F(y)\}\ x := y \oplus z\ \{\mathsf{E}(\mathcal{E}, x)\}$ if $y \neq z$

How to generate $E(\mathcal{E}, x)$?

Xor

(X4) $\{F(y)\}\ x := y \oplus z\ \{E(\mathcal{E}, x)\}$ if $y \neq z$

Counter

- (I1) $\{F(y)\}\ x := y + 1\ \{RCounter(x) \wedge E(\mathcal{E}, x)\}$
- (I2) $\{RCounter(y)\}\ x := y + 1\ \{RCounter(x) \wedge E(E, x)\}$

20 Rules

$x \xleftarrow{\$} \mathcal{U}$

(R1)
(R2)

$x = y||z$

(C1)
(C2)

$x := y + 1$

(I1)
(I2)
(I3)

(G1)
(G2)
(G3)
(G4)

$x := y \oplus z$

(X1)
(X2)
(X3)
(X4)

$x := \mathcal{E}(y)$

(B1)
(B2)
(B3)
(B4)
(B5)

How to prove that a Generic Encryption Mode is IND-CPA?

Theorem

Let $\mathcal{E}_M(m_1|\ldots|m_p, c_0|\ldots|c_p)$: **var** $\vec{x}$; c be a generic encryption mode,
Then $\mathcal{E}_M$ is IND-CPA secure, if
$\{\text{true}\}c \bigwedge_{i=0}^{i=p}\{\text{Indis}(\nu c_i; m_1, \ldots, m_p, c_0, \ldots, c_p)\}$ is valid.

Example: CBC

$\mathcal{E}_{CBC}(m_1|m_2|m_3, IV|c_1|c_2|c_3)$

var IV, z_1, z_2, z_3;

$IV \xleftarrow{\$} \mathcal{U}$;	$\text{Indis}(\nu IV; \text{Var}) \wedge F(IV)$	(R1)
$z_1 := IV \oplus m_1$;	$\text{Indis}(\nu IV; \text{Var} - z_1) \wedge \text{E}(\mathcal{E}, z_1, IV)$	(X2)(X4)
$c_1 := \mathcal{E}(z_1)$;	$\text{Indis}(\nu IV; \text{Var} - z_1)$	(B2)
	$\wedge \text{Indis}(\nu c_1; \text{Var}) \wedge F(c_1)$	(B1)
$z_2 := c_1 \oplus m_2$;	$\text{Indis}(\nu IV; \text{Var} - z_1)$	(G1)
	$\wedge \text{Indis}(\nu c_1; \text{Var} - z_2) \wedge \text{E}(\mathcal{E}, z_2)$	(X2)(X4)
$c_2 := \mathcal{E}(z_2)$;	$\text{Indis}(\nu IV; \text{Var} - z_1) \wedge \text{Indis}(\nu c_1; \text{Var} - z_2)$	(B2)
	$\wedge Indis(\nu c_2; \text{Var}) \wedge F(c_2)$	(B1)
$z_3 := c_2 \oplus m_3$;	$\text{Indis}(\nu IV; \text{Var} - z_1) \wedge \text{Indis}(\nu c_1; \text{Var} - z_2)$	(G1)
	$\wedge \text{Indis}(\nu c_2; \text{Var} - z_3) \wedge \text{E}(\mathcal{E}, z_3)$	(X2)(X4)
$c_3 := \mathcal{E}(z_3)$;	$\text{Indis}(\nu IV; \text{Var} - z_1) \wedge \text{Indis}(\nu c_1; \text{Var} - z_2)$	(B2)
	$\wedge Indis(\nu c_3; \text{Var}) \wedge \text{Indis}(\nu c_2; \text{Var} - z_3)$	(B1)

Prototype

Implementation of a backward analysis in 1000 lines of Ocaml.

Examples

- CBC, FBC, OFB CFB are proved IND-CPA
- ECB and variants our tool fails: precondition is not true

All examples are immediate (less than one second)

Summary

- Generic Encryption Mode
- New predicats
- Hoare Logic for proving generic encryption mode IND-CPA
- Ocaml Prototype

Future Works

- Hybrid encryption
- using LSFR (Dual Encryption Mode)
- using Hash function
- using mathematics (GMC)
- IND-CCA ?
  Desai 2000: New Paradigms for Constructing Symmetric Encryption Schemes Secure against Chosen-Ciphertext Attack
- Considering : For loops

Thank you for your attention

Questions ?