On Unique Decomposition of Processes in the Applied π-Calculus

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(Unique) Parallel Decomposition in Process Algebras

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  where $P_1, \ldots, P_n$ are "prime", i.e. cannot be decomposed into nontrivial processes?
- Is this decomposition unique?

Applications

- Provides a normal form
- Gives a cancellation result, i.e.

$$P|Q = P|R \Rightarrow Q = R$$

- Provides a maximally parallelized version of a given program
- Can be used to verify the equivalence of two processes [GM92]

Previous Results

Unique decomposition results exist

- for the Calculus of Communicating Systems (CCS) [Mil89] by Moller and Milner [MM93, Mol89]:
  - finite processes w. interleaving or parallel composition w.r.t. strong bisimilarity
  - finite processes w. parallel composition w.r.t. weak bisimilarity
- for Basic Parallel Processes (BPP) [Chr93]:
  - normed processes w. interleaving or parallel composition w.r.t. strong bisimilarity
- for ordered monoids by Luttik and van Oostrom:
  - if the calculus satisfies certain properties, the result for strong bisimilarity follows directly [LvO05]
  - can be extended to weak bisimilarity [Lut12]

The Applied π-Calculus [AF01]

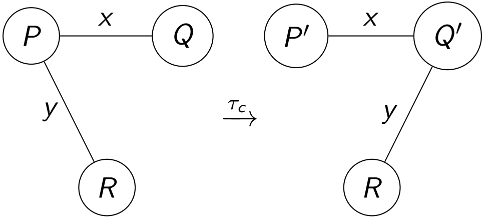
- an "impure" variant of the π-Calculus
- designed for the verification of cryptographic protocols
- features an *equational theory* to model cryptographic primitives:

$$dec(enc(m, k), k) = m$$

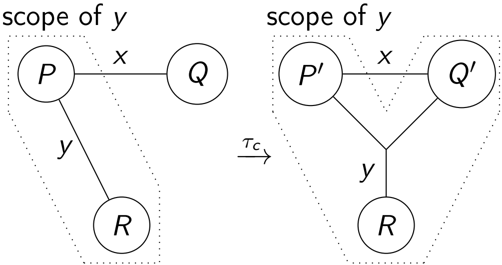
- and *active substitutions* $\{^M/_x\}$, a non-zero element that exhibits no transitions
- allows *channel* or *link passing* (sometimes also called *mobility*) and *scope extrusion*

Channel/Link passing

Consider three parallel processes P, Q and R. P and Q synchronize using an internal reduction τ_c:
$P|Q|R \xrightarrow{\tau_c} P'|Q'|R$



Scope extrusion



Plan

1. Introduction
2. The Applied π-Calculus
3. Results
   - Strong Bisimilarity
   - Weak Bisimilarity
4. Conclusion

Plan

Syntax

Plain processes:

$P, Q :=$	plain processes
0	null process
$P \mid Q$	parallel composition
$!P$	replication
$\nu n.P$	name restriction ("new")
$\texttt{if } M = N \texttt{ then } P \texttt{ else } Q$	conditional (M, N terms)
$\texttt{in}(u, x).P$	message input
$\texttt{out}(u, M).P$	message output

Syntax Cont'd

Active/extended processes:

$A, B, P, Q :=$	active processes
P	plain process
$A \vert B$	parallel composition
$\nu n.A$	name restriction
$\nu x.A$	variable restriction
$\{^M/_x\}$	active substitution

Plan

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Strong Labeled Bisimilarity

Definition (Strong Labeled Bisimilarity ($\sim_l$))

Strong labeled bisimilarity is the largest symmetric relation $\mathcal{R}$ on closed active processes, such that $A \mathcal{R} B$ implies:

1. $A \approx_s B$,
2. if $A \rightarrow A'$, then $B \rightarrow B'$ and $A' \mathcal{R} B'$ for some B',
3. if $A \xrightarrow{\alpha} A'$ and $fv(\alpha) \subseteq \text{dom}(A)$ and $bn(\alpha) \cap fn(B) = \emptyset$, then $B \xrightarrow{\alpha} B'$ and $A' \mathcal{R} B'$ for some B'.

Strongly Parallel Prime

Definition (Strongly Parallel Prime)

A closed process P is *strongly parallel prime*, if

- $P \not\sim_l 0$ and
- for any two closed processes Q and R such that $P \sim_l Q|R$, we have $Q \sim_l 0$ or $R \sim_l 0$.

Example 1

Example

Consider the following process:

$$P_{ex} \quad = \quad \nu k.\nu l.\nu m.\nu d.\left(\{^{l}/_{y}\}\,|\mathtt{out}(c, enc(n,k))|\right.$$
$$\left.\mathtt{out}(d,m)|\mathtt{in}(d,x).\mathtt{out}(c,x)\right)$$

We can decompose P_{ex} as follows:

$$P_{ex} \quad \sim_l \quad (\nu l.\,\{^{l}/_{y}\})|(\nu k.\mathtt{out}(c, enc(n,k)))|$$
$$(\nu d.(\nu m.\mathtt{out}(d,m)|\mathtt{in}(d,x).\mathtt{out}(c,x)))$$

Example 2

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Consider $!P$ for a process $P \not\sim_l 0$.
By definition $!P = P|!P$, hence $!P$ is not prime.
There is no decomposition into prime factors.

Existence of Factorization

Theorem (Existence of Factorization)

Any closed normed process P can be expressed as the parallel product of strong parallel primes, i.e.

$$P \sim_l P_1 | \ldots | P_n$$

where for all $1 \leq i \leq n$ P_i is strongly parallel prime.

Proof by induction on the norm and the size of the domain.

Uniqueness of Factorization

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The strong parallel factorization of a closed normed process P is unique (up to $\sim_l$).

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Proof idea:

- *Proof by induction* on the norm of P, and inside each case by induction on the size of the domain
- Each prime factor can either perform a *transition*, or has a *non-empty domain*
- A transition might not always be *norm-reducing* since processes can be infinite, but there is always a norm-reducing one
- Suppose the existence of two different factorizations, and show that this leads to a *contradiction*

Uniqueness of Factorization, Proof Cont'd

Four cases: A process with

- no transition and empty domain: unique factorization 0.
- no transition but non-empty domain: apply a *restriction* on part of the domain to hide all factors but one. Exploit the *induction hypothesis*
- empty domain, but transitions: execute a *transition* and apply the induction hypothesis.
  - *Problem:* an internal reduction can fuse factors using scope extrusion.
  - *Solution:* Whenever possible, choose a visible transition.
  - No visible transition $\Rightarrow$ processes cannot fuse using an internal reduction, since this would mean they synchronized on a public channel $\Rightarrow$ visible transitions exist.
- non-empty domain and transitions: combine the above two

Plan

Weak Labeled Bisimilarity

Definition (Weak Labeled Bisimilarity ($\approx_l$) [AF01])

(Weak) Labeled Bisimilarity is the largest symmetric relation $\mathcal{R}$ on closed active processes, such that $A \mathcal{R} B$ implies:

1. $A \approx_s B$,
2. if $A \rightarrow A'$, then $B \rightarrow^* B'$ and $A' \mathcal{R} B'$ for some B',
3. if $A \xrightarrow{\alpha} A'$ and $fv(\alpha) \subseteq \text{dom}(A)$ and $bn(\alpha) \cap fn(B) = \emptyset$, then $B \rightarrow^* \xrightarrow{\alpha} \rightarrow^* B'$ and $A' \mathcal{R} B'$ for some B'.

Weakly Parallel Prime

Definition (Weakly Parallel Prime)

A closed extended process P is *weakly parallel prime*, if

- $P \not\approx_l 0$ and
- for any two closed processes Q and R such that $P \approx_l Q|R$, we have $Q \approx_l 0$ or $R \approx_l 0$.

Example 3

Example

Consider

$$P = \nu a.(out(a,m)|(in(a,x).(!in(b,y)))|in(a,x))$$

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$$P = \nu a.(out(a,m)|(in(a,x).(!in(b,y)))|in(a,x))$$

We have

$$P \to \nu a.(!in(b,y)|in(a,x)) \approx_l !in(b,y)$$

and

$$P \to \nu a.(in(a,x).(!in(b,y))) \approx_l 0.$$

Example 3

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Consider

$$P = \nu a.(out(a, m)|(in(a, x).(!in(b, y)))|in(a, x))$$

We have

$$P \to \nu a.(!in(b, y)|in(a, x)) \approx_l !in(b, y)$$

and

$$P \to \nu a.(in(a, x).(!in(b, y))) \approx_l 0.$$

Thus $P \approx_l P|P$, hence we have no unique decomposition.

Existence of Factorization

Theorem (Existence of Factorization)

Any closed finite active process P can be expressed as the parallel product of parallel primes, i.e.

$$P \approx_l P_1 | \ldots | P_n$$

where for all $1 \leq i \leq n$ P_i is weakly parallel prime.

Uniqueness of Factorization

Theorem (Uniqueness of Factorization)

The parallel factorization of a closed finite process P is unique (up to $\approx_l$*).*

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The parallel factorization of a closed finite process P is unique (up to $\approx_l$).

Proof idea:

- Show the following statement: Any closed finite processes P and Q with $P \approx_l Q$ have the same factorization (up to $\approx_l$)
- *Induction* on the sum of the total depth of both factorizations, and in each case on the size of the domain
- Suppose the existence of two different factorizations and show this leads to a *contradiction*

Proof of Uniqueness of Factorization, Cont'd

- *Same structure* as the proof for strong bisimilarity
- Problem:
  - each transition can be simulated using *several* internal reductions
  - can affect several factors, and prime factors could *fuse* using scope extrusion
- Solution:
  - choose transitions that decrease the visible depth by *exactly one*
  - A synchronization of two factors uses *at least two* visible actions $\Rightarrow$ the resulting processes cannot be bisimilar any more

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Conclusion and future work

- Two unique decomposition results for subsets of the Applied π-Calculus:
  - closed finite processes w.r.t. weak labeled bisimilarity
  - closed normed processes w.r.t. strong labeled bisimilarity
- Future work:
  - Replication (Bang) "!":
    - First result by Hirschkoff and Pous [HP10] for a subset of CCS with top-level replication: *seed* Q of a process P of least size (in terms of prefixes) whose number of replicated components is maximal
    - Similar result for the Restriction-Free-π-Calculus (i.e. no "ν") – full calculus remains an open question
  - Find an (efficient) algorithm computing the unique decomposition of a process?

Thank you for your attention!

Questions?

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Summary of results

Type of Process	Strong Bisimilarity ($\sim_l$)	Weak Bisimilarity ($\approx_l$)
finite	Theorem 5	Theorem 10
normed	Theorem 5	Counterexample 9
general	Counterexample 4	Counterexample 4

Depth of processes

Definition (Total Depth)

Let $\text{length}_t : (\mathbf{Act} \cup \mathbf{Int})^* \mapsto \mathbb{N}$ be a function where $\text{length}_t(\epsilon) = 0$ and $\text{length}_t(\mu w) = 1 + \text{length}_t(w)$. The *total depth* $|P|_t \in (\mathbb{N} \cup \{\infty\})$ of a closed process P is defined as follows:

$$|P|_t = \sup \left\{ \text{length}_t(w) : P \xrightarrow{w} P' \not\rightarrow, w \in (\mathbf{Act} \cup \mathbf{Int})^* \right\}$$

Norm of a process

Definition (Norm of a Process)

Let $\mathsf{length}_n : (\mathbf{Act} \cup \mathbf{Int})^* \mapsto \mathbb{N}$ be a function where $\mathsf{length}_n(\epsilon) = 0$ and $\mathsf{length}_n(\mu w) = \begin{cases} 1 + \mathsf{length}_n(w) & \text{if } \mu \neq \tau_c \\ 2 + \mathsf{length}_n(w) & \text{if } \mu = \tau_c \end{cases}$
The norm $\mathcal{N}(P) \in (\mathbb{N} \cup \{\infty\})$ of a closed process P is defined as follows:

$$\mathcal{N}(P) = \inf \left\{ \mathsf{length}_n(w) : P \xrightarrow{w} P' \not\rightarrow, w \in (\mathbf{Act} \cup \mathbf{Int})^* \right\}$$

Some properties

- $P = Q|R$ implies $|P|_v = |Q|_v + |R|_v$
- $P = Q|R$ implies $|P|_t = |Q|_t + |R|_t$
- $P = Q|R$ implies $\mathcal{N}(P) = \mathcal{N}(Q) + \mathcal{N}(R)$
- $|P|_v \leq |P|_t$